One-year and ultimate correlations in dependent claims run-off triangles *

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Abstract: We investigate a bottom-up risk aggregation applied by insurance companies facing reserve risk from multiple lines of business. Since the required risk capitals should be calculated in different time horizons, depending on the regulatory or reporting regime (Solvency II vs IFRS 17), we study correlation coefficients which one should use for future calendar years in one-year horizon and in ultimate horizon when applying the variance-covariance aggregation formula given stand-alone risk capitals. We consider a multivariate version of a Hertig's lognormal model and we derive analytical formulas for the ultimate correlation and the one-year correlations in future calendar years. Our first conclusion is that we cannot use one correlation coefficient to aggregate stand-alone risk capitals for lines of business in different time horizons and in different calendar years, i.e. correlations for a bottom-up risk aggregation depend on the time horizon and the future calendar year where the risk emerges. Our second conclusion is that the one-year correlation in the next calendar year is higher than the ultimate correlation, i.e. an insurance company tends to over-estimate its risk adjustment if it uses one-year correlations from Solvency II for a bottom-up risk aggregation in IFRS 17.

Keywords: Ultimate correlation, one-year correlations, claims reserving, Hertig's lognormal model, Solvency II, IFRS 17.

*This research is financially supported with grant NCN 2018/31/B/HS4/02150

1 Introduction

In this paper we investigate a bottom-up risk aggregation applied by insurance companies facing reserve risk from multiple lines of business. In a bottom-up approach, an insurance company first determines risk capitals for each line of business and next aggregates the risk capitals by applying the variance-covariance formula in order to determine a diversified risk capital at the level of a company (which is next allocated back to lines of business to reflect diversification benefits between the lines of business). Such an approach to establish a diversified risk capital for an insurance company and quantify diversification benefits is specified in Solvency II Standard Formula and often used in actuarial practice. Clearly, correlations play an important role in a bottom-up risk aggregation. Depending on the regulatory or reporting regime, the required risk capitals should be calculated in different time horizons. In Solvency II regulatory regime, insurance companies calculate so-called solvency capital requirements and risk margins (risk capitals for future calendar years in one-year time horizons). In IFRS 17 reporting standard, insurance companies calculate so-called risk adjustments (risk capitals in ultimate time horizon). The goal of this paper is to study correlation coefficients which one should use for future calendar years in one-year horizon and in ultimate horizon when applies the variance-covariance aggregation formula given stand-alone risk capitals for lines of business. Questions about correlations in different time horizons have recently gained more attention among actuaries, as insurance companies now have to quantify, at the same time, their risks in one-year and ultimate time horizons under Solvency II and IFRS 17.

The importance of measuring risk in different time horizons in claims reserving problems has been deeply discussed in Wüthrich et al. (2019) and Wüthrich, Merz (2015). The authors focus on the mean square errors of prediction and a decomposition of the ultimate risk into a sequence of the one-year risks in future calendar years. However, they do not discuss correlation coefficients implied by the mean square errors of prediction in different time horizons, which are the key parameters of interest when an insurance company applies a bottom-up aggregation of stand-alone risk capitals.

We consider a multivariate version of a Hertig's lognormal model from Merz et al. (2012). We study dependent run-off triangles for multiple lines of business and derive correlations coefficients in different time horizons which should be used for a bottom-up risk aggregation. In our model, we can investigate general dependence structures for claims development processes within and between lines of business. In particular, we focus on the following three types of dependence structures, which are usually studied in non-life claims reserving:

- Cell-wise dependence between run-off triangles, see e.g. Braun (2004), Shi and Frees (2011), Iturria et al. (2021).
- Cell-wise dependence between run-off triangles and calendar year dependence within run-off triangles, see e.g. Shi et al. (2012),
- Cell-wise dependence between run-off triangles, calendar year dependence and vanishing dependence between calendar years within run-off triangles, see e.g. Shi et al. (2012).

Merz et al. (2012) derive the mean square error of prediction of the one-year loss in the next calendar year and the mean square error of prediction of the ultimate loss. We extend their results and we also derive the mean square errors of prediction of the one-year loss in all future calendar years, which are needed to calculate risk margin in Solvency II. Based on the mean square errors of prediction, calculated in different time horizons and in different calendar years, we introduce the notion of ultimate correlation and one-year correlations in future calendar years between lines of business. By their definitions, these correlations serve as the inputs to the variance-covariance aggregation formula which should be applied to determine a diversified risk capital from stand-alone risk capitals in ultimate and one-year time horizons. We derive analytical formulas for the ultimate correlation and the one-year correlations in future calendar years in our multivariate Hertig's lognormal model. The formulas allow us to study the values of the correlation coefficients for a bottom-up risk aggregation in different time horizons and in different calendar years in the multivariate Hertig's lognormal model with a specified dependence structure between and within run-off triangles. They also allow us to switch (calibrate) the one-year correlation from the ultimate correlation and vice-verse. For some special cases of the claims development processes in our claims development model, we derive more explicit (and simpler) formulas for the ultimate correlation and the one-year correlations in future calendar years. Let us remark that Merz et al. (2012) also study correlation coefficients implied by the mean square errors of prediction, but the authors use a different definition of correlations and they are interested in correlations between accident years.

Our first key conclusion is that we cannot use one correlation coefficient to aggregate stand-alone risk capitals for lines of business in different time horizons and in different calendar years. The most straightforward approach in actuarial practice would be to take the one-year correlations between lines of business from Solvency II Standard Formula, which by their purpose should be used to calculate the diversified solvency capital requirement for the next calendar year (the regulatory capital in Solvency II), and apply the same correlations in all future calendar years to derive the future diversified solvency capital requirements for the risk margin, as well as to correlate risk capitals in ultimate time horizon to calculate the risk adjustment for IFRS 17 reporting. We demonstrate in this paper that this is not the correct approach for quantifying the true diversification effects in different time horizons. We show that, in general, correlations for a bottom-up risk aggregation depend on the time horizon and the future calendar year where the risk emerges.

Our second key conclusion is that, under some assumptions, the one-year correlation in the next calendar year is higher than the ultimate correlation. As a consequence, if an insurance company uses one-year correlations from Solvency II for risk aggregation in IFRS 17, it tends to over-estimate the diversified risk capital (the risk adjustment at the level of a company). This conclusion is confirmed in our numerical examples under less restrictive assumptions. The message from this paper to actuaries is that ultimate and one-year correlations are different and these differences should not be neglected a priori as they may have an impact on calculations performed in Solvency II and IFRS 17.

To the best of our knowledge, this is one of the first papers in actuarial science which studies correlations coefficients between losses in different time horizons. We would like to remark that the notions of one-year correlation and ultimate correlation have been also recently introduced by El Alami et al. (2022). El Alami et al. (2022) consider a different actuarial model with additive cash flows from elliptical distribution with special dependence structures (in particular, with time dependence between cash flows). The authors only investigate a one-year correlation in the next calendar year, and they do not discuss one-year correlations in future calendar years. Interestingly, El Alami et al. (2022) also show that, in their model, the one-year correlation in the next calendar year is higher than the ultimate correlation.

This paper is structured as follows. In section 2, we introduce a multivariate Hertig's lognormal model. In Section 3 we define the ultimate correlation and the one-year correlations in future calendar years. The two key relations between the correlations are derived in Section 4. Numerical examples are presented in Section 5. All proofs can be found in Section 6.

2 The multivariate model of claims development

We use a multivariate version of Hertig's lognormal model of claims development from Merz et al. (2012). Below, we present some key results on the claims development processes in lines of business, which we need to derive ultimate and one-year correlations between lines of business. For details on the multivariate Hertig's lognormal model, we refer to Merz et al. (2012).

Let us consider N lines of business which are labelled by $n = 1, \dots, N$. We denote accident years by $i \in \{1, \dots, I\}$ and development years by $j \in \{0, \dots, J\}$. As always, we assume that $I \geq J + 1$ and all claims are settled within $J + 1$ development years. To define cell-wise correlations between claims development processes in the lines of business, we assume that all lines of business have the same number of historical accident years and development years, hence I and J do not depend on n. The cumulative payments from accident year i after development year j for line of business n are denoted by $C_{i,j,n}$.

In our multivariate Hertig's lognormal model, we assume that

- The development of claims is given by the process:

$$C_{i,j,n} = C_{i,j-1,n} e^{\xi_{i,j,n}}, \quad (i,j,n) \in \big\{1, \dots, I\big\} \times \big\{0, \dots, J\big\} \times \big\{1, \dots, N\big\}, \tag{2.1}$$

  and we set w.l.o.g. $C_{i,-1,n} = 1$.

We define the vectors:

$$\boldsymbol{\xi}_{i,j} = \big(\xi_{i,j,1}, \dots, \xi_{i,j,N}\big)' \in \mathbb{R}^N, \quad \boldsymbol{\xi}_i = \big(\boldsymbol{\xi}'_{i,0}, \dots, \boldsymbol{\xi}'_{i,J}\big)' \in \mathbb{R}^a, \quad \boldsymbol{\xi} = \big(\boldsymbol{\xi}'_1, \dots, \boldsymbol{\xi}'_I\big)' \in \mathbb{R}^d,$$

where $a = N(J+1)$ and $d = aI$, and we assume that

- Conditionally, given $\boldsymbol{\Theta} \in \mathbb{R}^a$, the random vector $\boldsymbol{\xi}$ has a multivariate Gaussian distribution with fixed positive-definite covariance matrix $\Sigma \in \mathbb{R}^{d \times d}$ and conditional expected values $\mathbb{E}\big[\boldsymbol{\xi}_i | \boldsymbol{\Theta}\big] = \boldsymbol{\Theta}$ for all $i \in \{1, \dots, I\}$,

- The parameter $\boldsymbol{\Theta}$ has a multivariate Gaussian distribution with prior mean $\boldsymbol{\mu} \in \mathbb{R}^a$ and positive-definite prior covariance matrix $T \in \mathbb{R}^{a \times a}$.

In this paper we are particularly interested in the following dependence structures between the claims run-offs:

- Dependence A: Portfolio correlation and calendar year independence, see Merz et al. (2012):

$$\begin{aligned} Cov\big[\xi_{i,j,n}, \xi_{l,z,m} | \boldsymbol{\Theta}\big] &= \sigma_{j,n}\sigma_{z,m}\rho \mathbf{1}\{(i,j) = (l,z), (i,j,n) \neq (l,z,m)\}, \\ Cov\big[\xi_{i,j,n}, \xi_{l,z,m} | \boldsymbol{\Theta}\big] &= \sigma^2_{i,n} \mathbf{1}\{(i,j) = (l,z), (i,j,n) = (l,z,m)\}, \end{aligned}$$

- Dependence B: Portfolio and calendar year correlation, see Merz et al. (2012):

$$
\begin{aligned}
Cov\left[\xi_{i,j,n}, \xi_{l,z,m} | \mathbf{\Theta}\right] &= \sigma_{j,n}\sigma_{z,m}\rho \mathbf{1}\{i+j=l+z, (i,j,n) \neq (l,z,m)\}, \\
Cov\left[\xi_{i,j,n}, \xi_{l,z,m} | \mathbf{\Theta}\right] &= \sigma_{i,n}^2 \mathbf{1}\{(i,j)=(l,z), (i,j,n) = (l,z,m)\},
\end{aligned}
$$

- Dependence C: Portfolio, calendar year and trend correlation, see Shi et al. (2012):

$$
\begin{aligned}
Cov\left[\xi_{i,j,n}, \xi_{l,z,m} | \mathbf{\Theta}\right] &= \sigma_{j,n}\sigma_{z,m}\rho^h \mathbf{1}\{i+j-l-z=h-1, (i,j,n) \neq (l,z,m)\}, \\
&\quad h = 1, 2, \ldots, \\
Cov\left[\xi_{i,j,n}, \xi_{l,z,m} | \mathbf{\Theta}\right] &= \sigma_{i,n}^2 \mathbf{1}\{(i,j)=(l,z), (i,j,n) = (l,z,m)\}.
\end{aligned}
$$

In dependence structure A we assume that there is only a cell-wise correlation between the claims development noises $\boldsymbol{\xi}$ in the loss triangles of the lines of business. In dependence structure B we additionally assume that there is a correlation between the claims development noises $\boldsymbol{\xi}$ arising in the same calendar year within and between the loss triangles. Finally, in dependence structure C we assume, in addition to A and B, that there is a correlation between the claims development noises $\boldsymbol{\xi}$ in all calendar years, within and between the loss triangles, yet the correlation decreases for more distant calendar years.

As far as the parameters' uncertainty is concerned, we consider the following cases:

- Parameters' uncertainty: The uncertainties of the a priori parameters' estimates for the lines of business and the development years are independent from each other, see Merz et al. (2012):

$$
T = \tau^2 \mathbf{1}_{a \times a}, \tag{2.2}
$$

  where $\tau > 0$ and $\mathbf{1}_{a \times a}$ denotes an identity matrix of dimension $a \times a$,

- No parameters' uncertainty: we set $\tau = 0$ in (2.2).

In Bayesian setting, the diagonal structure of T implies that actuaries make independent decisions about the parameters' estimates based on their a priori knowledge. We remark that we only consider the parameters' uncertainty related to the expected value of $\boldsymbol{\xi}$, as in Merz et al. (2012). If we would like to measure uncertainty related to the specification of the covariance matrix of $\boldsymbol{\xi}$, then a full simulation model has to be run in the spirit of Shi et al. (2012).

The unconditional distribution of $\boldsymbol{\xi}$ is known, see e.g. Theorem 3.1 from Merz et al. (2012).

Theorem 2.1. *The unconditional distribution of* $\boldsymbol{\xi}$ *is Gaussian with the moments:*

$$\mathbb{E}\big[\boldsymbol{\xi}\big] = A\boldsymbol{\mu}, \quad cov\big[\boldsymbol{\xi}\big] = \Sigma + ATA',$$

where $A = (\mathbf{1}_{a\times a}, \ldots, \mathbf{1}_{a\times a})' \in \mathbb{R}^{d\times a}$.

Let $t = 1, 2, \ldots$ denote a calendar year. We introduce $\mathcal{D}_t = \big\{(i,j,n) \in \big\{1,\ldots,I\big\} \times \big\{0,\ldots,J\big\} \times \big\{1,\ldots,N\big\}, i+j \leq t\big\}$. The set $\mathcal{D}_t$ contains indices of the cumulative payments $C_{i,j,n}$, equivalently, the indices of the claims development noises $\xi_{i,j,n}$, which have been observed at the end of calendar year t for all lines of business. For observations from a single line of business, we use $\mathcal{D}_t^n$ for $n = 1,\ldots,N$. We also introduce the filtration:

$$\mathcal{F}_t = \sigma\big\{C_{i,j,n} : (i,j,n) \in \mathcal{D}_t\big\},$$

which describes the information available after t calendar years from all lines of business. To describe the information available from a single line of business, we use $\mathcal{F}_t^n$.

Next, we define matrices $\mathcal{P}_{\mathcal{D}_t} : \mathbb{R}^d \mapsto \mathbb{R}^{|\mathcal{D}_t|}$ and $\mathcal{P}_{\mathcal{D}_t^c} : \mathbb{R}^d \mapsto \mathbb{R}^{|\mathcal{D}_t^c|}$ such that

$$\boldsymbol{\xi} \mapsto \boldsymbol{\xi}^{\mathcal{D}_t} = \mathcal{P}_{\mathcal{D}_t}\boldsymbol{\xi}, \quad \boldsymbol{\xi} \mapsto \boldsymbol{\xi}^{\mathcal{D}_t^c} = \mathcal{P}_{\mathcal{D}_t^c}\boldsymbol{\xi}.$$

The vector $\boldsymbol{\xi}$ contains the Gaussian noises describing the whole claims development process for all lines of business. The vector $\boldsymbol{\xi}^{\mathcal{D}_t}$ contains the Gaussian noises from the claims development process which have been observed at the end of the calendar year t, and the vector $\boldsymbol{\xi}^{\mathcal{D}_t^c}$ contains the Gaussian noises from the claims development process which will be observed after the calendar year t. In exactly the same way, we also define

$$\boldsymbol{\xi}^n, \quad \boldsymbol{\xi}^{\mathcal{D}_t^{n,c}}, \quad \boldsymbol{\xi}^{\mathcal{D}_t^n}, \quad \mathcal{P}_{\mathcal{D}_t^n}, \quad \mathcal{P}_{\mathcal{D}_t^{n,c}}, \tag{2.3}$$

for a single line of business, $n = 1,\ldots,N$.

The goal in claims reserving is to derive the conditional distribution of $\boldsymbol{\xi}^{\mathcal{D}_t^c}$ given $\boldsymbol{\xi}^{\mathcal{D}_t}$. This distribution was derived in Theorem 3.4. by Merz et al. (2012).

Theorem 2.2. *The conditional distribution of* $\boldsymbol{\xi}^{\mathcal{D}_t^c}$ *given* $\boldsymbol{\xi}^{\mathcal{D}_t}$ *is multivariate Gaussian with the conditional mean*

$$\boldsymbol{\mu}_{\mathcal{D}_t^c}^{post} = \mathbb{E}\big[\boldsymbol{\xi}^{\mathcal{D}_t^c}\big|\boldsymbol{\xi}^{\mathcal{D}_t}\big] = \mathcal{P}_{\mathcal{D}_t^c}A\boldsymbol{\mu} + \mathcal{Q}_{\mathcal{D}_t,\mathcal{D}_t^c}\Big(\boldsymbol{\xi}^{\mathcal{D}_t} - \mathcal{P}_{\mathcal{D}_t}A\boldsymbol{\mu}\Big),$$

and conditional covariance matrix

$$S_{\mathcal{D}_t^c}^{post} = cov\big[\boldsymbol{\xi}^{\mathcal{D}_t^c}\big|\boldsymbol{\xi}^{\mathcal{D}_t}\big] = \mathcal{P}_{\mathcal{D}_t^c}S\mathcal{P}_{\mathcal{D}_t^c}' - \mathcal{Q}_{\mathcal{D}_t,\mathcal{D}_t^c}\mathcal{P}_{\mathcal{D}_t}S\mathcal{P}_{\mathcal{D}_t^c}'$$

where

$$\begin{aligned} A &= (\mathbf{1}_{a\times a}, \ldots, \mathbf{1}_{a\times a})' \in \mathbb{R}^{d\times a}, \\ \mathcal{Q}_{\mathcal{D}_t,\mathcal{D}_t^c} &= \mathcal{P}_{\mathcal{D}_t^c} S \mathcal{P}'_{\mathcal{D}_t} \Big(\mathcal{P}_{\mathcal{D}_t} S \mathcal{P}'_{\mathcal{D}_t}\Big)^{-1}, \\ S &= cov\big[\boldsymbol{\xi}\big] = \Sigma + ATA'. \end{aligned}$$

Let us recall that $\mathbf{1}_{a\times a} \in \mathbb{R}^{a\times a}$ denotes an identity matrix of dimension $a \times a$.

Remark 2.1. We can note that

$$cov\big[\xi_{i,j,n}, \xi_{l,z,m}\big] = \mathbb{E}\Big[cov\big[\xi_{i,j,n}, \xi_{l,z,m}\big|\boldsymbol{\Theta}\big]\Big] + cov\big[\theta_{j,n}, \theta_{z,m}\big].$$

We can deduce that under dependence structures A and B and without parameters' uncertainty ($\tau = 0$ in (2.2)), the vector $\boldsymbol{\xi}^{\mathcal{D}_t^c}$ is independent of $\boldsymbol{\xi}^{\mathcal{D}_t}$ since

$$cov\big[\xi_{i,j,n}, \xi_{l,z,m}\big] = 0, \quad for \quad i + j \neq l + z.$$

For this particular case, we will derive more explicit results for ultimate and one-year correlations. □

In the sequel, we will select elements of the vector $\boldsymbol{\xi}^{\mathcal{D}_t^c}$. Let us define a vector $\mathbf{e}_{t|i,j\in\mathcal{J},n}$ of dimension $|\mathcal{D}_t^c| \times 1$, which contains zeros and ones, such that

$$\mathbf{e}'_{t|i,j\in\mathcal{J},n}\boldsymbol{\xi}^{\mathcal{D}_t^c} = \sum_{j=t-i+1}^{J} \xi_{i,j,n}\mathbf{1}\{j \in \mathcal{J}\}.$$

We will use the notation:

$$\mathbf{e}'_{t|i,j,n}\boldsymbol{\xi}^{\mathcal{D}_t^c} = \xi_{i,j,n}\mathbf{1}\{t-i+1 \leq j \leq J\}, \quad \mathbf{e}'_{t|i,j\leq M,n}\boldsymbol{\xi}^{\mathcal{D}_t^c} = \sum_{j=t-i+1}^{M} \xi_{i,j,n}\mathbf{1}\{t-i+1 \leq M\},$$

where the indicators guarantee that we choose indices $j \in \mathcal{J}$ such that $j \in [t-i+1, M]$, otherwise $\mathbf{e}'_{t|i,j\in\mathcal{J},n}$ only contains zeros.

3 Risk measures and implied correlations for bottom-up risk aggregation

We measure the reserve risk at the end of calendar year $t = I$. In the next subsections, we derive risk measures in ultimate and one-year horizon and define ultimate and one-year correlations between lines of business implied by the risk measures.

3.1 Ultimate risk and ultimate correlation

For accident year i such that $i + J > t$, the ultimate liability (the ultimate cumulative payments) for the accident year and line of business n is given by

$$C_{i,J,n} = C_{i,t-i,n} e^{\sum_{j=t-i+1}^{J} \xi_{i,j,n}} = C_{i,t-i,n} e^{\mathbf{e}'_{t|i,j \leq J,n} \boldsymbol{\xi}^{\mathcal{D}_t^c}}. \tag{3.1}$$

For other accident years, the claims have been fully developed and these accident years are no longer investigated in the claims reserving problem. The total ultimate liability for all accident years and all lines of business is given by

$$C_J = \sum_{i=t-J+1}^{J} \sum_{n=1}^{N} C_{i,J,n}.$$

The best estimate of the ultimate liability at the end of calendar year t is defined with

$$\hat{C}_{i,J,n}^{t} = \mathbb{E}\big[C_{i,J,n} | \mathcal{F}_t\big], \quad i + J > t.$$

By Theorem 2.2 (and Theorem 5.1), we get the formula:

$$\begin{aligned} \hat{C}_{i,J,n}^{t} &= C_{i,t-i,n} e^{\sum_{j=t-i+1}^{J} \mathbb{E}\left[\xi_{i,j,n} | \mathcal{F}_t\right] + \frac{1}{2} \sum_{j,l=t-i+1}^{J} cov\left[\xi_{i,j,n}, \xi_{i,l,n} | \mathcal{F}_t\right]} \\ &= C_{i,t-i,n} e^{\mathbf{e}'_{t|i,j \leq J,n} \boldsymbol{\mu}_{\mathcal{D}_t^c}^{post} + \frac{1}{2} \mathbf{e}'_{t|i,j \leq J,n} S_{\mathcal{D}_t^c}^{post} \mathbf{e}_{t|i,j \leq J,n}}, \end{aligned} \tag{3.2}$$

for accident years such that $i + J > t$.

We investigate the ultimate loss projected from the end of calendar year t. The ultimate loss for accident year i and line of business n is given by

$$L_{i,n}^{Ult,t} = C_{i,J,n} - \hat{C}_{i,J,n}^{t},$$

for accident years such that $i + J > t$. The total ultimate loss for all accident years and all lines of business is given by

$$L^{Ult,t} = \sum_{n=1}^{N} \sum_{i=t-J+1}^{I} L_{i,n}^{Ult,t}.$$

In claims reserving, the risk of future payments is usually measured with the mean square error of prediction. Based on Merz et al. (2012) and Wüthrich, Merz (2015), we know that the mean square error of prediction in our model coincides with the variance measure. The variance risk measure in our Bayesian claims reserving model quantifies both the process error and the estimation error. We have the following result on the ultimate risk of the ultimate loss, which is Theorem 4.3 from Merz et al. (2012).

Theorem 3.1. *We have the formula for the ultimate risk measure:*

$$
\begin{aligned}
Var\left[L^{Ult,t}\middle|\mathcal{F}_t\right] &= \sum_{n,m=1}^{N}\sum_{i,l=t-J+1}^{I} cov\left[L_{i,n}^{Ult,t}, L_{l,m}^{Ult,t}\middle|\mathcal{F}_t\right] \\
&= \sum_{n,m=1}^{N}\sum_{i,l=t-J+1}^{I} cov\left[C_{i,J,n}, C_{l,J,m}\middle|\mathcal{F}_t\right] \\
&= \sum_{n,m=1}^{N}\sum_{i,l=t-J+1}^{I} \hat{C}_{i,J,n}^{t}\hat{C}_{l,J,m}^{t}\left(e^{\sum_{j=t-i+1}^{J}\sum_{z=t-l+1}^{J} cov\left[\xi_{i,j,n},\xi_{l,z,m}|\mathcal{F}_t\right]} - 1\right) \\
&= \sum_{n,m=1}^{N}\sum_{i,l=t-J+1}^{I} \hat{C}_{i,J,n}^{t}\hat{C}_{l,J,m}^{t}\left(e^{\mathbf{e}'_{t|i,j\leq J,n} S_{\mathcal{D}_t^c}^{post}\mathbf{e}_{t|l,j\leq J,m}} - 1\right). \qquad (3.3)
\end{aligned}
$$

If we want to measure the ultimate risk for single line of business n, we should calculate variance of the ultimate loss for that particular line of business conditional on $\mathcal{F}_t^n$. To do this calculation, we should use the inner summation in (3.3) for n reformulated by using the vectors and the matrices from (2.3).

We now introduce the notion of ultimate correlation. The ultimate correlation represents the correlation which should be used for a bottom-up aggregation of risk capitals in ultimate time horizon. The ultimate correlation is implied by the variance risk measures from Theorem 3.1.

Definition 3.1. *For two lines of business, denoted by $n = 1, 2$, the implied ultimate correlation is derived from the relation:*

$$
\begin{aligned}
Var\left[L^{Ult,t}\middle|\mathcal{F}_t\right] &= Var\left[L_1^{Ult,t}\middle|\mathcal{F}_t^1\right] + Var\left[L_2^{Ult,t}\middle|\mathcal{F}_t^2\right] \\
&\quad +2\sqrt{Var\left[L_1^{Ult,t}\middle|\mathcal{F}_t^1\right]Var\left[L_2^{Ult,t}\middle|\mathcal{F}_t^2\right]}\, corr\left[L_1^{Ult,t}, L_2^{Ult,t}\middle|\mathcal{F}_t\right], \qquad (3.4)
\end{aligned}
$$

where

$$
L_n^{Ult,t} = \sum_{i=t-J+1}^{I} L_{i,n}^{Ult,t}, \quad n = 1, 2.
$$

Remark 3.1. In general, we have

$$
Var\left[L_n^{Ult,t}\middle|\mathcal{F}_t\right] \neq Var\left[L_n^{Ult,t}\middle|\mathcal{F}_t^n\right], \quad n = 1, 2. \qquad (3.5)
$$

As a consequence, $corr\left[L_1^{Ult,t}, L_2^{Ult,t}\middle|\mathcal{F}_t\right]$ is not a true correlation coefficient due to different filtrations on the left and the right hand side of (3.4). □

Remark 3.2. We can deduce that the equality in (3.5) holds for dependence structures A and B and our special case of the parameters' uncertainty given by a diagonal matrix T (see (2.2)). For these cases, the ultimate correlation (3.4) is a true correlation coefficient. For dependence structure C, we have inequality in (3.5) and the formula (3.4) does not give us a true correlation (we may end up with values outside $[-1, 1]$). □

There are different possible definitions of the correlation coefficient for a bottom-up risk aggregation. E.g. Merz et al. (2012) use a different definition. It all depends on the aggregation method applied by an insurance company and the definition of the stand-alone risk capitals for lines of business. We believe that (3.4) is the closest to practical applications where an actuary quantifies the risk in separate lines of business using the information available for stand-alone lines of business and quantifies the total risk using the information for all lines of business.

Formula (3.4) is valid for any dependence structure between the lines of business and any parameters' uncertainty in our multivariate Hertig's lognormal model. We can derive explicit results in some special and important cases.

Theorem 3.2. *Let the implied ultimate correlation be given with*

$$corr\left[L_1^{Ult,t}, L_2^{Ult,t} \middle| \mathcal{F}_t\right] = \frac{A}{\sqrt{B_1}\sqrt{B_2}}.$$

- *For dependence structure A and no parameters' uncertainty, we have:*

$$\begin{aligned} A &= \sum_{i=t-J+1}^{I} \hat{C}_{i,J,n}^t \hat{C}_{i,J,m}^t \left(e^{\sum_{j=t-i+1}^{J} \sigma_{j,n}\sigma_{j,m}\rho} - 1 \right) \\ &\approx \sum_{i=t-J+1}^{I} \hat{C}_{i,J,n}^t \hat{C}_{i,J,m}^t \sum_{j=t-i+1}^{J} \sigma_{j,n}\sigma_{j,m}\rho, \\ B_n &= \sum_{i=t-J+1}^{I} \left(\hat{C}_{i,J,n}^t\right)^2 \left(e^{\sum_{j=t-i+1}^{J} \sigma_{j,n}^2} - 1 \right) \\ &\approx \sum_{i=t-J+1}^{I} \left(\hat{C}_{i,J,n}^t\right)^2 \sum_{j=t-i+1}^{J} \sigma_{j,n}^2, \quad n = 1, 2, \end{aligned}$$

where the approximations hold for small $(\sigma_{j,n})_{j=0}^J$ for $n = 1, 2$.

- *For dependence structure B and no parameters' uncertainty, we have:*

$$
\begin{aligned}
A &= \sum_{i,l=t-J+1}^{I} \hat{C}^t_{i,J,n}\hat{C}^t_{l,J,m}\Big(e^{\sum_{j=t-i+1}^{J\wedge(J+l-i)}\sigma_{j,n}\sigma_{i+j-l,m}\rho}-1\Big),\\
&\approx \sum_{i,l=t-J+1}^{I} \hat{C}^t_{i,J,n}\hat{C}^t_{l,J,m}\sum_{j=t-i+1}^{J\wedge(J+l-i)}\sigma_{j,n}\sigma_{i+j-l,m}\rho\\
B_n &= \sum_{i=t-J+1}^{I}\big(\hat{C}^t_{i,J,n}\big)^2\Big(e^{\sum_{j=t-i+1}^{J\wedge(J+l-i)}\sigma^2_{j,n}}-1\Big)\\
&\quad+\sum_{i,l=t-J+1,\ i\neq l}^{I}\hat{C}^t_{i,J,n}\hat{C}^t_{l,J,n}\Big(e^{\sum_{j=t-i+1}^{J\wedge(J+l-i)}\sigma_{j,n}\sigma_{i+j-l,n}\rho}-1\Big)\\
&\approx \sum_{i=t-J+1}^{I}\big(\hat{C}^t_{i,J,n}\big)^2\sum_{j=t-i+1}^{J}\sigma^2_{j,n}\\
&\quad+\sum_{i,l=t-J+1,\ i\neq l}^{I}\hat{C}^t_{i,J,n}\hat{C}^t_{l,J,n}\sum_{j=t-i+1}^{J\wedge(J+l-i)}\sigma_{j,n}\sigma_{i+j-l,n}\rho,\qquad n=1,2,
\end{aligned}
$$

where the approximations hold for small $(\sigma_{j,n})_{j=0}^{J}$ *for* $n=1,2$.

As already discussed in the paper, the purpose of the ultimate correlation is to use it in a bottom-up aggregation of stand-alone risk capitals in ultimate time horizon. In this approach, an insurance company specifies risk capitals in ultimate time horizon for (two) lines of business $RC^{Ult}_{t,1}$ and $RC^{Ult}_{t,2}$ and applies the variance-covariance aggregation formula to derive the diversified risk capital:

$$
\sqrt{\big(RC^{Ult}_{t,1}\big)^2+\big(RC^{Ult}_{t,2}\big)^2+2\cdot RC^{Ult}_{t,1}\cdot RC^{Ult}_{t,2}\cdot\rho^{Ult}_t}, \tag{3.6}
$$

where ρ^{Ult}_t denotes the calibrated ultimate correlation. In many practical applications, the risk capitals in (3.6) are (closely) related to variance risk measures of ultimate losses, which justifies the choice of our correlation coefficient (3.4) in the aggregation formula (3.6). Traditionally, actuaries have been measuring risk in ultimate time horizon. Recently, the new IFRS17 accounting standard forces insurance companies to calculate risk adjustments which should be related to a risk measure in ultimate time horizon.

3.2 One-year risks and one-year correlations

We now study a sequence of the best estimates of the ultimate liability at the end of future calendar years $t, t+1, \ldots$, which are needed to define one-year risks in future calendar years.

Let $k = 0, 1, \ldots$. We define $\hat{C}^{t+k}_{i,J,n} = \mathbb{E}\big[C_{i,J,n} | \mathcal{F}_{t+k}\big]$ for accident years such that $i + J \geq t + k$. We can derive the formula for the best estimate of the ultimate liability at the end of any calendar year $t + k$ viewed from the end of calendar year t.

Proposition 3.1. *For $k = 0, 1, \ldots$, we have the formula for the best estimate of the ultimate liability:*

$$\begin{aligned} \hat{C}^{t+k}_{i,J,n} &= C_{i,t-i,n} e^{\sum_{j=t-i+1}^{J} \mathbb{E}\left[\xi_{i,j,n} | \mathcal{F}_{t+k}\right]} \\ &\cdot e^{+\frac{1}{2}\sum_{j,l=t-i+1}^{J} cov\left[\xi_{i,j,n}, \xi_{i,l,n} | \mathcal{F}_t\right] - \frac{1}{2}\sum_{j,l=t-i+1}^{J} cov\left[\mathbb{E}[\xi_{i,j,n}|\mathcal{F}_{t+k}], \mathbb{E}[\xi_{i,l,n}|\mathcal{F}_{t+k}] | \mathcal{F}_t\right]} \\ &= C_{i,t-i,n} e^{\mathbf{p}'_{t|i,k,n} \boldsymbol{\xi}^{\mathcal{D}^c_t} + r_{t|i,k,n}}, \quad i + J \geq t + k, \end{aligned} \tag{3.7}$$

where

$$\begin{aligned} \mathbf{p}'_{t|i,k,n} &= \mathbf{e}'_{t|i,j\leq t-i+k,n} + \mathbf{e}'_{t+k|i,j\leq J,n} \mathcal{Q}_{\mathcal{D}_{t+k}, \mathcal{D}^c_{t+k}} \mathcal{P}_{\mathcal{D}_{t+k}} \mathcal{P}'_{\mathcal{D}^c_t} \mathbf{1}\{i + J > t + k\}, \\ r_{t|i,k,n} &= \big(\mathbf{e}'_{t|i,j\leq J,n} - \mathbf{p}'_{t|i,k,n}\big) \boldsymbol{\mu}^{post}_{\mathcal{D}^c_t} + \frac{1}{2} \mathbf{e}'_{t|i,j\leq J,n} S^{post}_{\mathcal{D}^c_t} \mathbf{e}_{t|i,j\leq J,n} - \frac{1}{2} \mathbf{p}'_{t|i,k,n} S^{post}_{\mathcal{D}^c_t} \mathbf{p}_{t|i,k,n}. \end{aligned}$$

Formula (3.7) is an extension of Lemma 5.1 from Merz et al. (2012). By (3.7) we can study the one-year risk in all future calendar years $t + k$, whereas Merz et al. (2012) only study the one-year risk in the next calendar year $t + 1$. We can observe that for $k = 0$ the formula (3.7), as expected, reduces to (3.2), since $\mathbf{e}'_{t|i,j\leq t+k-i,n}$ and $\mathcal{P}_{\mathcal{D}_t} \mathcal{P}'_{\mathcal{D}^c_t}$ contain zeros, hence $\mathbf{p}'_{t|i,k,n}$ contains zeros.

Let us investigate the one-year loss in calendar year $t + k + 1$ projected from the end of calendar year $t + k$, for $k = 0, 1, \ldots, J - 1$. The one-year loss in calendar year $t + k + 1$ for accident year i and line of business n is given by

$$L^{1YR,t+k+1}_{i,n} = \hat{C}^{t+k+1}_{i,J,n} - \hat{C}^{t+k}_{i,J,n},$$

for accident years such that $i + J > t + k$. Other accident years have been fully developed by that calendar time. The total one-year loss in calendar year $t + k + 1$ for all accident years and lines of business is given by

$$L^{1YR,t+k+1} = \sum_{n=1}^{N} \sum_{i=t+k-J+1}^{I} L^{1YR,t+k+1}_{i,n}.$$

The risk of the one-year loss is usually measured, as the risk of the ultimate loss, with the mean squared error of prediction, which again agrees with the variance measure in our model. In order to quantify the one-year risk for the next calendar year, we should calculate the conditional variance:

$$Var\Big[L^{1YR,t+k+1}\Big|\mathcal{F}_{t+k}\Big], \tag{3.8}$$

since the one-year risk in calendar $t+k+1$ is projected from the end of calendar year $t+k$. Since we measure the risk at the end of calendar year $t = I$ and we can only project the risk from the end of calendar year t, we calculate the conditional expected value of (3.8) given $\mathcal{F}_t$:

$$\mathbb{E}\Big[Var\Big[L^{1YR,t+k+1}\Big|\mathcal{F}_{t+k}\Big]\Big|\mathcal{F}_t\Big],$$

as a projection of the one-year risk in the future calendar year.

There is an obvious relation between the ultimate loss and the one-year losses in future calendar years:

$$L^{Ult,t} = \sum_{k=0}^{J-1} L^{1YR,t+k+1}.$$

From Wüthrich, Merz (2015), we also have the following crucial property for the ultimate risk and the one-year risks:

$$Var\big[L^{Ult,t}\big|\mathcal{F}_t\big] = \sum_{k=0}^{J-1} Var\big[L^{1YR,t+k+1}\big|\mathcal{F}_t\big] = \sum_{k=0}^{J-1} \mathbb{E}\Big[Var\big[L^{1YR,t+k+1}\big|\mathcal{F}_{t+k}\big]\mathcal{F}_t\Big], \tag{3.9}$$

which shows how the ultimate risk can be split into the one-year risks in future calendar years under variance as the risk measure. In particular, the one-year losses in future calendar years are not correlated. The decomposition (3.9) does not hold in general, but it holds in Bayesian claims reserving models, and in our claims reserving model.

We can now derive the one-year risk of the one-year loss in all future calendar years.

Theorem 3.3. *We have the formula for the one-year risk measures:*

$$
\begin{aligned}
&\mathbb{E}\Big[Var\big[L^{1YR,t+k+1}\big|\mathcal{F}_{t+k}\big]\mathcal{F}_t\Big] = Var\Big[L^{1YR,t+k+1}\Big|\mathcal{F}_t\Big] \\
&= \sum_{n,m=1}^{N}\sum_{i,l=t+k-J+1}^{I} cov\Big[L_{i,n}^{1YR,t+k+1}, L_{l,m}^{1YR,t+k+1}\Big|\mathcal{F}_t\Big] \\
&= \sum_{n,m=1}^{N}\sum_{i,l=t+k-J+1}^{I} cov\Big[\hat{C}_{i,J,n}^{t+k+1} - \hat{C}_{i,J,n}^{t+k}, \hat{C}_{l,J,m}^{t+k+1} - \hat{C}_{l,J,m}^{t+k}\Big|\mathcal{F}_t\Big] \\
&= \sum_{n,m=1}^{N}\sum_{i,l=t+k-J+1}^{I} \Big(cov\Big[\hat{C}_{i,J,n}^{t+k}\hat{C}_{l,J,m}^{t+k}\Big|\mathcal{F}_t\Big] + cov\Big[\hat{C}_{i,J,n}^{t+k+1}\hat{C}_{l,J,m}^{t+k+1}\Big|\mathcal{F}_t\Big] \\
&\quad -2cov\Big[\hat{C}_{i,J,n}^{t+k}\hat{C}_{l,J,m}^{t+k+1}\Big|\mathcal{F}_t\Big]\Big) \\
&= \sum_{n,m=1}^{N}\sum_{i,l=t+k-J+1}^{I} \hat{C}_{i,J,n}^{t}\hat{C}_{l,J,m}^{t}\Big(e^{\sum_{j=t-i+1}^{J}\sum_{z=t-l+1}^{J} cov\left[\mathbb{E}[\xi_{i,j,n}|\mathcal{F}_{t+k}],\mathbb{E}[\xi_{l,z,n}|\mathcal{F}_{t+k}]|\mathcal{F}_t\right]} \\
&\quad + e^{\sum_{j=t-i+1}^{J}\sum_{z=t-l+1}^{J} cov\left[\mathbb{E}[\xi_{i,j,n}|\mathcal{F}_{t+k+1}],\mathbb{E}[\xi_{l,z,n}|\mathcal{F}_{t+k+1}]|\mathcal{F}_t\right]} \\
&\quad -2e^{\sum_{j=t-i+1}^{J}\sum_{z=t-l+1}^{J} cov\left[\mathbb{E}[\xi_{i,j,n}|\mathcal{F}_{t+k}],\mathbb{E}[\xi_{l,z,n}|\mathcal{F}_{t+k+1}]|\mathcal{F}_t\right]}\Big) \\
&= \sum_{n,m=1}^{N}\sum_{i,l=t+k-J+1}^{I} \hat{C}_{i,J,n}^{t}\hat{C}_{l,J,m}^{t}\Big(e^{\mathbf{p}'_{t|i,k,n}S_{\mathcal{D}_t^c}^{post}\mathbf{p}_{t|l,k,m}} + e^{\mathbf{p}'_{t|i,k+1,n}S_{\mathcal{D}_t^c}^{post}\mathbf{p}_{t|l,k+1,m}} \\
&\quad -2e^{\mathbf{p}'_{t|i,k,n}S_{\mathcal{D}_t^c}^{post}\mathbf{p}_{t|l,k+1,m}}\Big).
\end{aligned} \tag{3.10}
$$

Let us remark that for $k = 0$, the formula (3.10) reduces to Theorem 5.2 from Merz et al. (2012) since the vector $\mathbf{p}'_{t|i,0,n}$ contains zeros. If we want to measure the one-year risk for single line of business n, we should calculate variance of the one-year loss for that particular line of business conditional on $\mathcal{F}_t^n$. To do this calculation, we should use the inner summation in (3.10) for n reformulated by using the vectors and the matrices from (2.3).

We now define one-year correlations in future calendar years till the liability's run-off. The one-year correlations represent the correlations which should be used for a bottom-up aggregation of risk capitals in one-year time horizon in future calendar years. The one-year correlations are implied by the variance risk measures from Theorem 3.3.

Definition 3.2. *For two lines of business, denoted by $n = 1, 2$, the implied one-year correlation in calendar year $t + k + 1$ is derived from the relation:*

$$
\begin{aligned}
&Var\Big[L^{1YR,t+k+1}\Big|\mathcal{F}_t\Big] = Var\Big[L_1^{1YR,t+k+1}\Big|\mathcal{F}_t^1\Big] + Var\Big[L_2^{1YR,t+k+1}\Big|\mathcal{F}_t^2\Big] \\
&\quad +2\sqrt{Var\Big[L_1^{1YR,t+k+1}\Big|\mathcal{F}_t^1\Big]Var\Big[L_2^{1YR,t+k+1}\Big|\mathcal{F}_t^2\Big]}\,corr\Big[L_1^{1YR,t+k+1}, L_2^{1YR,t+k+1}\Big|\mathcal{F}_t\Big]
\end{aligned} \tag{3.11}
$$

In special cases, we can derive explicit results on the one-year correlations.

Theorem 3.4. *Let the implied one-year correlations in future calendar years be given with:*

$$corr\left[L_1^{1YR,t+k+1}, L_2^{1YR,t+k+1}\big|\mathcal{F}_t\right] = \frac{A^k}{\sqrt{B_1^k}\sqrt{B_2^k}}, \quad k = 0, 1, \ldots.$$

- *For dependence structure A and no parameter uncertainty, we have*

$$\begin{aligned} A^k &= \sum_{i=t+k-J+1}^{I} \hat{C}_{i,J,n}^t \hat{C}_{i,J,m}^t \left(e^{\sum_{j=t-i+1}^{t+k-i+1} \sigma_{j,n}\sigma_{j,m}\rho} - e^{\sum_{j=t-i+1}^{t+k-i} \sigma_{j,n}\sigma_{j,m}\rho}\right), \\ &\approx \sum_{i=t+k-J+1}^{I} \hat{C}_{i,J,n}^t \hat{C}_{i,J,m}^t \sigma_{t+k-i+1,n}\sigma_{t+k-i+1,m}\rho, \\ B_n^k &= \sum_{i=t+k-J+1}^{I} \left(\hat{C}_{i,J,n}^t\right)^2 \left(e^{\sum_{j=t-i+1}^{t+k-i+1} \sigma_{j,n}^2} - e^{\sum_{j=t-i+1}^{t+k-i} \sigma_{j,n}^2}\right) \\ &\approx \sum_{i=t+k-J+1}^{I} \left(\hat{C}_{i,J,n}^t\right)^2 \sigma_{t+k-i+1,n}^2, \end{aligned}$$

  where the approximations hold for small $(\sigma_{j,n})_{j=0}^J$ *for* $n = 1, 2$.

- *For dependence structure B and no parameter uncertainty, we have*

$$\begin{aligned} A^k &= \sum_{i,l=t+k-J+1}^{I} \hat{C}_{i,J,n}^t \hat{C}_{l,J,m}^t \left(e^{\sum_{j=t-i+1}^{(t+k-i+1)\wedge(J+l-i)} \sigma_{j,n}\sigma_{i+j-l,m}\rho} - e^{\sum_{j=t-i+1}^{(t+k-i)\wedge(J+l-i)} \sigma_{j,n}\sigma_{i+j-l,m}\rho}\right) \\ &\approx \sum_{i,l=t+k-J+1}^{I} \hat{C}_{i,J,n}^t \hat{C}_{l,J,m}^t \sigma_{t+k-i+1,n}\sigma_{t+k+1-l,m}\rho, \\ B_n^k &= \sum_{i=t+k-J+1}^{I} \left(\hat{C}_{i,J,n}^t\right)^2 \left(e^{\sum_{j=t-i+1}^{t+k-i+1} \sigma_{j,n}^2} - e^{\sum_{j=t-i+1}^{t+k-i} \sigma_{j,n}^2}\right) \\ &\quad + \sum_{i,l=t+k-J+1,\ i\neq l}^{I} \hat{C}_{i,J,n}^t \hat{C}_{l,J,n}^t \left(e^{\sum_{j=t-i+1}^{(t+k-i+1)\wedge(J+l-i)} \sigma_{j,n}\sigma_{i+j-l,n}\rho} - e^{\sum_{j=t-i+1}^{(t+k-i)\wedge(J+l-i)} \sigma_{j,n}\sigma_{i+j-l,n}\rho}\right) \\ &\approx \sum_{i=t+k-J+1}^{I} \left(\hat{C}_{i,J,n}^t\right)^2 \sigma_{t+k-i+1,n}^2 + \sum_{i,l=t+k-J+1,\ i\neq l}^{I} \hat{C}_{i,J,n}^t \hat{C}_{l,J,n}^t \sigma_{t+k-i+1,n}\sigma_{t+k+1-l,n}\rho, \end{aligned}$$

  where the approximations hold for small $(\sigma_{j,n})_{j=0}^J$ *for* $n = 1, 2$.

With formulas (3.3), (3.4), (3.10), (3.11), we can easily switch from the ultimate correlation to the one-year correlation for a given claims development process with a specified dependence structure of payments within and between loss triangles and parameters' uncertainty, and vice-verse, in the multivariate claims development model studied in this paper.

For other claims development models, our formulas should at least give an indication on how the correlations may change if we change the time horizon and the calendar year where the risk emerges.

The one-year correlation in the next calendar year (for $k = 0$) is used by insurance companies in a bottom-up risk aggregation in Solvency II to derive the regulatory capital. In practice, an insurance company specifies solvency capital requirements in one-year time horizon for the next calendar year for (two) lines of business $SCR^{1YR}_{t+1,1}$ and $SCR^{1YR}_{t+1,2}$ and applies the variance-covariance aggregation formula to derive the diversified solvency capital requirement:

$$\sqrt{\left(SCR^{1YR}_{t+1,1}\right)^2 + \left(SCR^{1YR}_{t+1,2}\right)^2 + 2 \cdot SCR^{1YR}_{t+1,1} \cdot SCR^{1YR}_{t+1,2} \cdot \rho^{1YR}_{t+1}}, \tag{3.12}$$

where ρ^{1YR}_{t+1} denotes the calibrated one-year correlation in the next calendar year. The solvency capital requirements in (3.12) are often (closely) related to variance risk measures of one-year losses, which justifies the choice of our correlation coefficients (3.11) in the aggregation formula (3.12). The key question is to what extent the ultimate correlation ρ^{Ult}_t from (3.6) may differ from the one-year correlation ρ^{1YR}_{t+1} from (3.12). We investigate this question in the next two sections.

The one-year correlations in future calendar years (for all $k \geq 0$) are important when we calculate risk margins in Solvency II. The risk margin is calculated as

$$\sum_{k=0}^{\infty} CoC \frac{SCR^{1YR}_{t+k+1}}{\left(1 + r_{t+k+1}\right)^{k+1}}, \tag{3.13}$$

where CoC is a cost of capital, r_{t+k+1} is the risk-free rate in calendar year $t + k + 1$ and SCR^{1YR}_{t+k+1} is the projected solvency capital requirement for calendar year $t + k + 1$. The solvency capital requirements can be determined with

$$\begin{aligned} SCR^{1YR}_{t+k+1} &= 3 \cdot \mathbb{E}\left[\sqrt{Var\left[L^{1YR,t+k+1} \middle| \mathcal{F}_{t+k}\right]} \middle| \mathcal{F}_t\right] \\ &\approx \; 3 \cdot \sqrt{\mathbb{E}\left[Var\left[L^{1YR,t+k+1} \middle| \mathcal{F}_{t+k}\right] \middle| \mathcal{F}_t\right]} = 3 \cdot \sqrt{Var\left[L^{1YR,t+k+1} \middle| \mathcal{F}_t\right]}, \end{aligned} \tag{3.14}$$

where the approximation is suggested by Wüthrich, Merz (2015) and the factor of 3 represents the quantile of order 99.5% of normal distribution assumed in Solvency II Standard Formula. In practice, many insurance companies project solvency capital requirements for future calendar years for each line of business and aggregate them with the variance-covariance aggregation formula to derive the diversified risk margin for a company with (3.13)-(3.14).

In this approach, one should use the one-year correlations in future calendar years for the aggregation of the SCRs projected for the future calendar years, which are likely to be different from the one-year correlation in the next calendar year. In the next two sections, we investigate patterns of the one-year correlations ρ^{1YR}_{t+k+1} in future calendar years, for $k = 0, 1, \ldots$, and inspect differences in the one-year correlations in future calendar years.

3.3 Two key relations between the ultimate correlation and the one-year correlations

In this section, we derive two important relations between the ultimate correlation and the one-year correlations in future calendar years. In the next section, we present more numerical results.

From (3.9), (3.4), (3.11), we can derive the equality:

$$\begin{aligned}
& Var\left[L_1^{Ult,t}\middle|\mathcal{F}_t^1\right] + Var\left[L_2^{Ult,t}\middle|\mathcal{F}_t^2\right] \\
& \quad +2\sqrt{Var\left[L_1^{Ult,t}\middle|\mathcal{F}_t^1\right]Var\left[L_2^{Ult,t}\middle|\mathcal{F}_t^2\right]}corr\left[L_1^{Ult,t}, L_2^{Ult,t}\middle|\mathcal{F}_t\right] \\
= & \quad Var\left[L^{Ult,t}\middle|\mathcal{F}_t\right] = \sum_{k=0}^{J-1} Var\left[L^{1YR,t+k+1}\middle|\mathcal{F}_t\right] \\
= & \quad \sum_{k=0}^{J-1}\Bigg(Var\left[L_1^{1YR,t+k+1}\middle|\mathcal{F}_t^1\right] + Var\left[L_2^{1YR,t+k+1}\middle|\mathcal{F}_t^2\right] \\
& \quad +2\sqrt{Var\left[L_1^{1YR,t+k+1}\middle|\mathcal{F}_t^1\right]Var\left[L_2^{1YR,t+k+1}\middle|\mathcal{F}_t^2\right]}corr\left[L_1^{1YR,t+k+1}, L_2^{1YR,t+k+1}\middle|\mathcal{F}_t\right]\Bigg).
\end{aligned}$$

Since (3.9) also holds for $L_1^{Ult,t}$ and $L_2^{Ult,t}$, we end up with the following relation between the risk measures:

$$\begin{aligned}
& \sqrt{Var\left[L_1^{Ult,t}\middle|\mathcal{F}_t^1\right]Var\left[L_2^{Ult,t}\middle|\mathcal{F}_t^2\right]}corr\left[L_1^{Ult,t}, L_2^{Ult,t}\middle|\mathcal{F}_t\right] \\
= & \sum_{k=0}^{J-1}\sqrt{Var\left[L_1^{1YR,t+k+1}\middle|\mathcal{F}_t^1\right]Var\left[L_2^{1YR,t+k+1}\middle|\mathcal{F}_t^2\right]}corr\left[L_1^{1YR,t+k+1}, L_2^{1YR,t+k+1}\middle|\mathcal{F}_t\right],
\end{aligned} \tag{3.15}$$

which allows us to state the first relation between the correlation coefficients.

Theorem 3.5. *Let $t = I$ and $(R_n^{t+k})_{k=0}^{J-1}$, for $n = 1, 2$, denote a risk run-off pattern for line of business n measured with*

$$R_n^{t+k+1} = \frac{\sqrt{Var\left[L_n^{1YR,t+k+1}\middle|\mathcal{F}_t^n\right]}}{\sqrt{Var\left[L_n^{Ult,t}\middle|\mathcal{F}_t^n\right]}} \in (0, 1).$$

We have the relation between the ultimate correlation and the one-year correlations:

$$corr\left[L_1^{Ult,t}, L_2^{Ult,t} \middle| \mathcal{F}_t\right] = \sum_{k=0}^{J-1} R_1^{t+k+1} R_2^{t+k+1} corr\left[L_1^{1YR,t+k+1}, L_2^{1YR,t+k+1} \middle| \mathcal{F}_t\right].$$

In particular, the one-year correlations in future calendar years cannot be all equal to the ultimate correlation.

The key conclusion from Theorem 3.5 is that we cannot use one correlation coefficient to aggregate stand-alone risk capitals for lines of business in different time horizons and in different calendar years. This is a very important conclusion for actuarial practice. The most straightforward approach in actuarial practice would be to take the one-year correlations between lines of business from Solvency II Standard Formula, which by their purpose should be used to calculate the diversified solvency capital requirement for the next calendar year (the regulatory capital in Solvency II), and apply the same correlations in all future calendar years to derive the future diversified solvency capital requirements used for the calculation of the risk margin, as well as to correlate the risk capitals in ultimate time horizon to calculate the risk adjustment for IFRS 17 reporting. Theorem 3.5 shows that this is not the correct approach for quantifying the true diversification effects in different time horizons and in different calendar years. We can conclude that correlations for a bottom-up risk aggregation depend on the time horizon and the future calendar year where the risk emerges.

Let us remark that Theorem 3.5 holds in any claims development model provided that the decomposition formula (3.9) holds. From Wüthrich, Merz (2015) we know that (3.9) holds approximately in Chain Ladder models, hence we expect that the key conclusion from Theorem 3.5 should also hold in Chain Ladder models.

In the next theorem, we directly compare the ultimate correlation with the one-year correlation in the next calendar year. Hence, we compare the correlation coefficients used for risk aggregation in Solvency II and IFRS 17. We assume an exponential pattern of volatility coefficients which is often observed in practice.

Theorem 3.6. *Let $t = I$. We assume that $\sigma_{j,n} = \sigma_{0,n} e^{-\alpha_n j}$ for $j = 0, \ldots, J$, and $e^{-\alpha_n J}$ is negligible, for $n = 1, 2$. Under the assumptions of Theorems 3.2 and 3.4, we have the relation between the ultimate correlation and the one-year correlation in the next calendar year:*

$$\begin{aligned} corr\left[L_1^{Ult,t}, L_2^{Ult,t} \middle| \mathcal{F}_t\right] &\approx \frac{\sqrt{1-e^{-2\alpha_1}}\sqrt{1-e^{-2\alpha_2}}}{1-e^{-(\alpha_1+\alpha_2)}} corr\left[L_1^{1YR,t+1}, L_2^{1YR,t+1} \middle| \mathcal{F}_t\right] \\ &\leq corr\left[L_1^{1YR,t+1}, L_2^{1YR,t+1} \middle| \mathcal{F}_t\right]. \end{aligned}$$

Our second key conclusion is that, under some assumptions, the one-year correlation in the next calendar year is higher than the ultimate correlation. As a consequence, if an insurance company uses one-year correlations from Solvency II for risk aggregation in IFRS 17, it tends to over-estimate the diversified risk capital (the risk adjustment at the level of a company). This conclusion is confirmed in our numerical examples under less restrictive assumptions on volatility patterns. We note that the reduction in the correlation coefficient when we switch from the one-year correlation to the ultimate correlation is large when the volatility coefficients in two lines of business have different tails convergence (α_1 is different from α_2) and is small when the volatility coefficients in two lines of business have similar tails convergence (α_1 is close to α_2). Interestingly, our result from Theorem 3.6 agrees with the result from El Alami et al. (2022), where the authors also show, in a different actuarial model, that the one-year correlation in the next calendar year is larger than the ultimate correlation.

4 Numerical examples

In this section, we present possible numerical values of the ultimate and one-year correlations which may be observed in practice. We consider two data sets with two lines of business. The first data set is taken from Merz et al. (2012) and it consists of two loss triangles for a motor third party liability and a general liability insurance. The second data set consists of two loss triangles for a motor vehicle liability insurance and fire and other damage to property insurance from the Polish market. For the first data set, we use the estimates of the parameters of the marginal Hertig's lognormal models from Merz et al. (2012). For the second data set, we use the historical loss triangles, estimate the parameters of the marginal Hertig's lognormal models and smooth them in late development periods with exponential functions. We do not estimate any dependence structure between the two lines of business in our data sets and we assume dependence structures A, B and C presented in Section 2. The goal of this section is to investigate how the ultimate and one-year correlations may differ from each other depending on dependence structure (A, B, C) driven by a single correlation coefficient ρ and parameters' uncertainty (independent parameters' uncertainty with volatility $\tau = 1$ and no parameters' uncertainty with $\tau = 0$). We choose $\rho = 0.2, 0.5, 0.8$.

The ultimate and one-year correlations are presented in Figures 1-2. We can observe that, in general, the correlations depend on the time horizon and the calendar year where the risk emerges, so we should not use one correlation coefficient in a bottom-up risk aggregation in different time horizons and in different calendar years. This observation clearly agrees with Theorem 3.5. For the data set from Merz et al. (2012), the one-year correlations in all future calendar years are almost equal to the ultimate correlation for dependence A. For dependence B and C, the one-year correlations in the first 7-8 calendar years stay at the similar level as the ultimate correlation and the one-year correlations in the calendar years after 7-8 years show a decreasing pattern. More interesting results are identified for the data set from the Polish market. We can see that the one-year correlation in the next calendar year is larger than the ultimate correlation in all cases. The one-year correlation in the next calendar year is larger than the ultimate correlation by 9% to 14%. For dependence A, the one-year correlations in all future calendar years are above the ultimate correlation, the one-year correlation slightly decreases in the first two calendar years and next it increases in the calendar years after the calendar year 3. For dependence B and C and no parameters' uncertainty, the one-year correlation increases in the future calendar years up to the calendar year 4 and next the one-year correlations show a decreasing pattern with respect to the calendar year, whereas for dependence B and C and parameters' uncertainty, the one-year correlations in future calendar years show a decreasing pattern with respect to the calendar year for all calendar years. For dependence B and C, the one-year correlations in future calendar years eventually fall below the ultimate correlation.

We now investigate the impact of a correlation in a bottom-up risk aggregation for calculating risk adjustments and risk margins. We use the data set from the Polish market. We assume the cost of capital is equal to 6% and the constant risk-free rate is equal to 3%. For the purpose of calculating the risk margin, we measure the risk with standard deviation of the one-year loss multiplied with 3 (which agrees with the approach from Solvency II Standard Formula). For the purpose of calculating the risk adjustment, we measure the risk with one standard deviation of the ultimate loss (which is close to the probability of fulfilling the liability at the level of 85% in ultimate time horizon, the confidence level targeted by insurance companies). The stand-alone risk capitals for the two lines of business are calculated with the variance measures presented in the paper and we aggregate these stand-alone risk capitals with various correlation coefficients. In Table 1, we present the following measures:

- RA_true - the risk adjustments obtained by using the ultimate correlation in the risk aggregation in ultimate time horizon,
- RA_avg - the risk adjustments obtained by using the average one-year correlation in the future calendar years in the risk aggregation in ultimate time horizon,

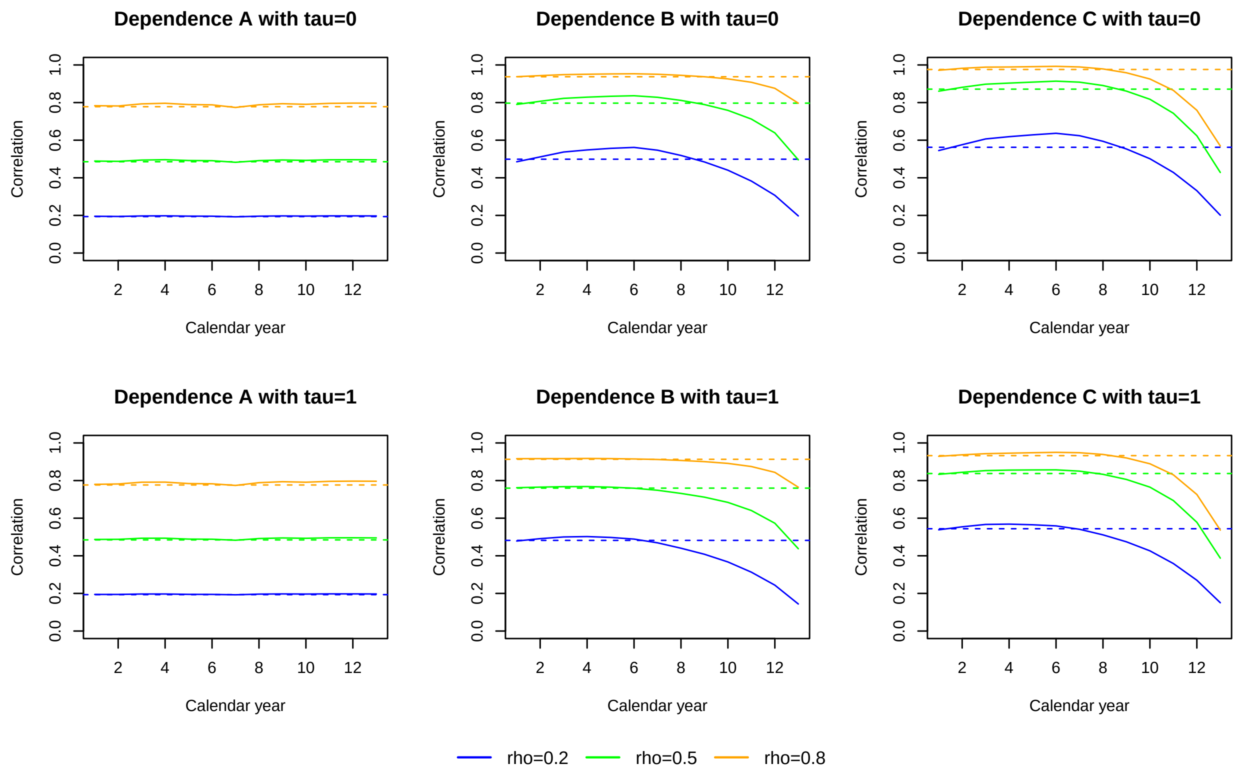

Figure 1: The one-year correlations in future calendar years (solid lines) and the ultimate correlation (dotted lines) - the loss triangles from Merz et al. (2012).

- RA_1YR - the risk adjustments obtained by using the one-year correlation in the next calendar year in the risk aggregation in ultimate time horizon,
- RM_true - the risk margins obtained by using the one-year correlations in the future calendar years in the risk aggregation in one-year time horizon,
- RM_1YR - the risk margins obtained by using the one-year correlation in the next calendar years in the risk aggregation in one-year time horizon,
- RM_ult - the risk margins obtained by using the ultimate correlation in the risk aggregation in one-year time horizon.

Even though we observe substantial differences in the ultimate and one-year correlations, see Figure 2, these differences turn out to be too small to lead to significant differences in the risk adjustments and the risk margins calculated in our example with a bottom-up risk aggregation with various correlations. The differences in the risk adjustments and the risk

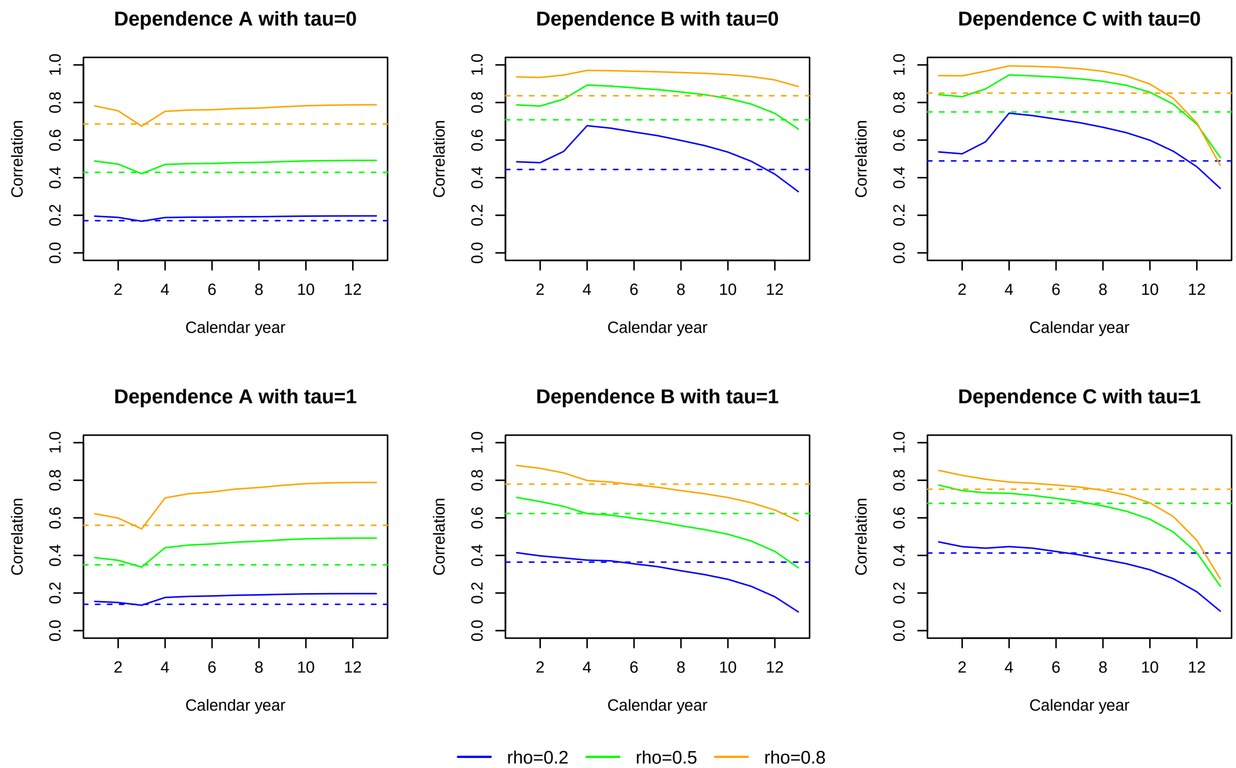

Figure 2: The one-year correlations in future calendar years (solid lines) and the ultimate correlation (dotted lines) - the loss triangles from line 4 and 7 from the Polish market.

margins are up to 3.7%, mostly $0.5-2\%$, compared to the true risk adjustment and the true risk margin calculated with proper correlation. If we calculate the risk adjustment with the one-year correlation in the next calendar year, instead of the ultimate correlation, we over-estimate the risk adjustment by $1-2\%$. This observation agrees with Theorem 3.6. If we calculate the risk margin with the ultimate correlation, instead of the one-year correlations in future calendar years, we under-estimate the risk margin by $0.5-1.5\%$. This result is intuitive since in our example the one-year correlations in the first calendar years are always above the ultimate correlation and the capital requirements in the first calendar years have a larger impact on the value of the risk margin due to discounting of the capital requirements.

The numerical differences in the risk adjustments and the risk margins in Table 1 calculated with various correlations (depending on the time horizon and the calendar year) may be disappointing at the first sight, but we should not conclude that risk adjustments and risk margins in IFRS 17 and Solvency II are indifferent to the correlation coefficient used in a bottom-up risk aggregation. It should be clearly pointed out that the results depend on the claims development model, the parameters of the marginal claims development processes and the duration of insurance liabilities. Hence, in a different example (in a different actuarial model), numerical differences in the ultimate and one-year correlations may be larger and risk adjustments and risk margin may be more sensitive to the correlation coefficient used in a bottom-up risk aggregation. Let us remark that El Alami et al. (2022) demonstrate that the difference in the risk adjustment calculated with the one-year correlation, instead of with the ultimate correlation, can reach 8%. However, El Alami et al. (2022) assumes that an insurer faces cash flows in 50 years and the cash flows are correlated between calendar years within lines of business. The authors explain that the difference in the risk adjustment of order of 8%, resulting from improper correlation in a bottom-up risk aggregation, can be attributed to dependence of cash flows between calendar years and long duration of cash flows. Please note that in our example, the liabilities in the two lines of business run-off after 13 years and the one-year losses are uncorrelated between future calendar years in each line of business.

τ	Dep. A	Dep. B	Dep. C	RA_true	RA_avg	RA_1YR	RM_true	RM_avg	RM_1YR	RM_ult
0.00	0.20	0.00	0.00	605.17	609.33	610.10	274.94	275.05	275.29	273.77
0.00	0.50	0.00	0.00	655.75	665.33	667.06	292.12	292.40	292.94	289.43
0.00	0.80	0.00	0.00	702.74	717.01	719.55	308.10	308.52	309.32	304.06
0.00	0.00	0.20	0.00	896.28	913.13	905.64	409.75	409.90	407.51	404.52
0.00	0.00	0.50	0.00	1,234.03	1,256.49	1,254.66	560.42	559.49	558.89	552.17
0.00	0.00	0.80	0.00	1,497.49	1,526.20	1,526.56	677.94	677.25	677.37	667.81
0.00	0.00	0.00	0.20	985.82	1,004.74	997.19	453.16	453.05	450.63	446.97
0.00	0.00	0.00	0.50	1,657.32	1,672.76	1,685.31	768.00	762.00	766.21	756.84
0.00	0.00	0.00	0.80	2,337.11	2,334.13	2,368.57	1,080.91	1,067.86	1,079.94	1,068.91
1.00	0.20	0.00	0.00	847.70	857.48	851.35	385.89	387.72	385.79	384.64
1.00	0.50	0.00	0.00	896.30	919.27	904.90	402.97	407.22	402.68	399.96
1.00	0.80	0.00	0.00	942.43	977.22	955.47	419.19	425.61	418.70	414.56
1.00	0.00	0.20	0.00	1,200.10	1,179.28	1,213.70	543.49	534.45	545.60	541.19
1.00	0.00	0.50	0.00	1,574.28	1,547.34	1,600.56	706.94	693.46	710.98	702.32
1.00	0.00	0.80	0.00	1,864.72	1,849.88	1,898.30	834.07	821.92	837.99	826.85
1.00	0.00	0.00	0.20	1,296.98	1,274.72	1,313.53	589.67	579.41	592.07	586.67
1.00	0.00	0.00	0.50	2,041.69	2,008.04	2,075.68	932.00	913.37	936.21	924.73
1.00	0.00	0.00	0.80	2,677.68	2,639.08	2,715.53	1,223.15	1,201.40	1,228.28	1,214.96

Table 1: The risk adjustments and the risk margins derived in a bottom-up risk aggregation with pre-specified correlation coefficients in dependence structures A, B and C and parameters' uncertainty with $\tau \in \{0, 1\}$ - the loss triangles from line 4 and 7 from the Polish market.

We should point out that the ultimate and one-year correlations, defined in this paper, depend on the claims development process assumed here. Clearly, the correlations derived in our multivariate Hertig's lognormal model cannot be used in a claims development model different from the one in which we calculate the risk measures (variance measures). Yet, we believe that the formulas presented here could be helpful to infer relations between ultimate and one-year correlations. Our numerical results do not clearly show the ultimate and the one-year correlations can differ to such an extent that they lead to significant differences in risk adjustments and risk margins if an incorrect correlation is used in a bottom-up risk aggregation. However, we believe that our results, which we present in this paper, should give a signal to actuaries that ultimate and one-year correlations are different and these differences should not be neglected a priori as they may have an impact on calculations performed in Solvency II and IFRS 17. The final impact of correlations on diversified risk capitals clearly depends on the claims development model assumed by the actuary.

5 Proofs

For readers' convenience, we first recall the well-know result on the distribution of a linear transformation of a multivariate Gaussian vector, which we apply to the vector $\boldsymbol{\xi}^{\mathcal{D}_t^c}$.

Theorem 5.1. *Let us define vectors* $\mathbf{a}$ *and* $\mathbf{b}$ *of dimension* $|\mathcal{D}_t^c| \times 1$*. The conditional distribution of* $\mathbf{a}'\boldsymbol{\xi}^{\mathcal{D}_t^c}$ *given* $\boldsymbol{\xi}^{\mathcal{D}_t}$ *is multivariate Gaussian with the conditional mean*

$$\mathbf{a}'\boldsymbol{\mu}_{\mathcal{D}_t^c}^{post},$$

and conditional covariance matrix

$$\mathbf{a}' S_{\mathcal{D}_t^c}^{post} \mathbf{a}.$$

We also have the formulas for the moments:

$$\begin{aligned}
\mathbb{E}\Big[e^{\mathbf{a}'\boldsymbol{\xi}^{\mathcal{D}_t^c}} \Big| \boldsymbol{\xi}^{\mathcal{D}_t}\Big] &= e^{\mathbf{a}'\boldsymbol{\mu}_{\mathcal{D}_t^c}^{post} + \frac{1}{2}\mathbf{a}' S_{\mathcal{D}_t^c}^{post}\mathbf{a}}, \\
cov\Big[e^{\mathbf{a}'\boldsymbol{\xi}^{\mathcal{D}_t^c}}, e^{\mathbf{b}'\boldsymbol{\xi}^{\mathcal{D}_t^c}} \Big| \boldsymbol{\xi}^{\mathcal{D}_t}\Big] &= \mathbb{E}\Big[e^{\mathbf{a}'\boldsymbol{\xi}^{\mathcal{D}_t^c}} \Big| \boldsymbol{\xi}^{\mathcal{D}_t}\Big] \cdot \mathbb{E}\Big[e^{\mathbf{b}'\boldsymbol{\xi}^{\mathcal{D}_t^c}} \Big| \boldsymbol{\xi}^{\mathcal{D}_t}\Big] \cdot \Big(e^{\mathbf{a}' S_{\mathcal{D}_t^c}^{post}\mathbf{b}} - 1\Big).
\end{aligned}$$

The proof of Theorems 3.2 and 3.4: We use Remark 2.1 and directly substitute the assumed covariance structures into (3.3) and (3.10). □

The proof of Proposition 3.1: We derive

$$\begin{aligned}
\hat{C}^{t+k}_{i,J,n} &= C_{i,t+k-i,n} e^{\left(\mathbf{e}'_{t+k|i,j\leq J,n}\boldsymbol{\mu}^{post}_{\mathcal{D}^c_{t+k}} + \frac{1}{2}\mathbf{e}'_{t+k|i,j\leq J,n} S^{post}_{\mathcal{D}^c_{t+k}} \mathbf{e}_{t+k|i,j\leq J,n}\right)\mathbf{1}\{i+J>t+k\}} \\
&= C_{i,t-i,n} e^{\sum_{j=t-i+1}^{t-i+k}\xi_{i,j,n} + \left(\mathbf{e}'_{t+k|i,j\leq J,n}\boldsymbol{\mu}^{post}_{\mathcal{D}^c_{t+k}} + \frac{1}{2}\mathbf{e}'_{t+k|i,j\leq J,n} S^{post}_{\mathcal{D}^c_{t+k}} \mathbf{e}_{t+k|i,j\leq J,n}\right)\mathbf{1}\{i+J>t+k\}} \\
&= C_{i,t-i,n} e^{\mathbf{e}'_{t|i,j\leq t-i+k,n}\boldsymbol{\xi}^{\mathcal{D}^c_t} + \mathbf{e}'_{t+k|i,j\leq J,n}\mathcal{Q}_{\mathcal{D}_{t+k},\mathcal{D}^c_{t+k}}\boldsymbol{\xi}^{\mathcal{D}_{t+k}}\mathbf{1}\{i+J>t+k\} + r_{t|i,k,n}} \\
&= C_{i,t-i,n} e^{\mathbf{e}'_{t|i,j\leq t-i+k,n}\boldsymbol{\xi}^{\mathcal{D}^c_t} + \mathbf{e}'_{t+k|i,j\leq J,n}\mathcal{Q}_{\mathcal{D}_{t+k},\mathcal{D}^c_{t+k}}\mathcal{P}_{\mathcal{D}_{t+k}}\mathcal{P}'_{\mathcal{D}^c_t}\mathbf{1}\{i+J>t+k\}\boldsymbol{\xi}^{\mathcal{D}^c_t} + r_{t|i,k,n}} \\
&= C_{i,t-i,n} e^{\mathbf{p}'_{t|i,k,n}\boldsymbol{\xi}^{\mathcal{D}^c_t} + r_{t|i,k,n}}.
\end{aligned}$$

In the derivation above, first, we use the estimate (3.2) after time $t+k$, the claims development process (2.1) and the definition of the conditional mean from Theorem 2.2. Next, we collect all $\mathcal{F}_{t+k}$-measurable terms and the residual term $r_{t|i,k,n}$ collects all $\mathcal{F}_t$-measurable terms. We notice that $\mathcal{P}'_{\mathcal{D}^c_t}\boldsymbol{\xi}^{\mathcal{D}^c_t}$ creates a vector of dimension $\mathbb{R}^d$ which contains $\xi_{i,j,n}$ for $(i,j,n) \in \mathcal{D}^c_t$ and sets $\xi_{i,j,n} = 0$ for $(i,j,n) \in \mathcal{D}_t$, which allows us to represent the $\mathcal{F}_{t+k}$-elements from $\boldsymbol{\xi}^{\mathcal{D}_{t+k}}$, which are not $\mathcal{F}_t$-measurable, with a linear transformation of $\boldsymbol{\xi}^{\mathcal{D}^c_t}$. Finally, $r_{t|i,k,n}$ is derived by the property that

$$\mathbb{E}\left[\hat{C}^{t+k}_{i,J,n}\middle|\mathcal{F}_t\right] = \hat{C}^t_{i,J,n},$$

which holds for any $k = 0, 1, \ldots$. For more details, we refer to Merz et al. (2012) where the formula for $k = 1$ is derived. □

The proof of Theorem 3.3: We use the definition of the loss, equation (3.9), Proposition 3.1, Theorem 5.1 and classical formulas for covariance. □

The proof of Theorem 3.5: The result follows from (3.15). □

The proof of Theorem 3.6: Dependence A. We substitute the exponential functions assumed for the volatility coefficients into the formula for the ultimate correlation from Theorem 3.2. We derive

$$\begin{aligned}
\sum_{j=t-i+1}^{J} \sigma_{j,1}\sigma_{j,2}\rho &= \sum_{j=t-i+1}^{J} \sigma_{0,1}\sigma_{0,2} e^{-\alpha_1 \cdot j} e^{-\alpha_2 \cdot j}\rho \\
&= \sigma_{0,1}\sigma_{0,2}\frac{e^{-(\alpha_1+\alpha_2)(t-i+1)} - e^{-(\alpha_1+\alpha_2)(J+1)}}{1-e^{-(\alpha_1+\alpha_2)}}\rho \\
&\approx \sigma_{0,1}\sigma_{0,2}\frac{e^{-(\alpha_1+\alpha_2)(t-i+1)}}{1-e^{-(\alpha_1+\alpha_2)}}\rho = \frac{\sigma_{t-i+1,1}\sigma_{t-i+1,2}\rho}{1-e^{-(\alpha_1+\alpha_2)}}.
\end{aligned}$$

In the same way, we handle $\sum_{j=t-i+1}^{J} \sigma_{j,n}^2$.

Dependence B. First, we derive

$$\begin{aligned} \sum_{j=t-i+1}^{J\wedge(J+l-i)} \sigma_{j,1}\sigma_{i+j-l,2}\rho &= \sum_{j=t-i+1}^{J\wedge(J+l-i)} \sigma_{0,1}\sigma_{0,2}e^{-\alpha_1\cdot j}e^{-\alpha_2\cdot(i+j-l)}\rho \\ &= \sigma_{0,1}\sigma_{0,2}\frac{e^{-\alpha_1\cdot(t-i+1)}e^{-\alpha_2\cdot(t+1-l)} - e^{-(\alpha_1+\alpha_2)\cdot(J\wedge(J+l-i)+1)-\alpha_2\cdot(i-l)}}{1-e^{-(\alpha_1+\alpha_2)}}\rho. \end{aligned}$$

Next, we can show that $-(\alpha_1+\alpha_2)\cdot(J\wedge(J+l-i)) - \alpha_2\cdot(i-l) \leq -(\alpha_1+\alpha_2)\cdot(J+1)$ for $t = I$ and $l - i \leq J - 1$ (as we consider i and l from $t - J + 1$ up to I to define the ultimate correlation). We conclude as above. □

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