Mean-Variance Portfolio Selection for a Non-life Insurance Company

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Abstract

We consider a collective insurance risk model with a compound Cox claim process, in which the evolution of a claim intensity is described by a stochastic differential equation driven by a Brownian motion. The insurer operates in a financial market consisting of a risk-free asset with a constant force of interest and a risky asset which price is driven by a Lévy noise. We investigate two optimization problems. The first one is the classical mean-variance portfolio selection. In this case the efficient frontier is derived. The second optimization problem, except the mean-variance terminal objective, includes also a running cost penalizing deviations of the insurer's wealth from a specified profit-solvency target which is a random process. In order to find optimal strategies we apply techniques from the stochastic control theory.

Keywords: Lévy diffusion financial market, compound Cox claim process, Hamilton-Jacobi-Bellman equation, Feynman-Kac representation, efficient frontier.

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1 Introduction

The idea of the classical mean-variance portfolio selection is to construct the best allocation of wealth among available assets in order to achieve the optimal trade-off between the expected return on an investment and its risk, measured by the variance. The mean-variance optimization problem was first introduced in Markowitz (1952) and this work has laid down foundations of the modern financial theory.

In this paper we extend the existing results concerning the mean-variance portfolio selection. The new features are a financial market with an asset driven by an infinite active Lévy process, a stochastic claim intensity leading to a compound Cox claim process and a mean-variance terminal objective with a running cost. To best our knowledge the optimization in this framework is taken for the first time. The goal is to derive, in our new general model, an investment strategy, verify its optimality and identify the efficient frontier. Another contribution is to find a classical solution of the corresponding Hamilton-Jacobi-Bellman equation. We arrive at a quadratic solution with coefficients depending on the time variable and the level of a claim intensity. The existence of a classical solution, and its form, is not surprising, however, it seems to appear for the first time in the control literature.

Optimal investment problems for non-life insurers have recently gained a lot of attention, see among others, Browne (1995), Schmidli (2002), Yang, Zhang (2005), Schäl (2005), Taksar (2000), Højgaard, Taksar (2004). The objectives adopted in these papers are the ultimate ruin probability or the expected value of dividends paid until ruin, which are the most common ones in the actuarial literature. This is in contrast with the objective applied widely in the financial literature, which is the expected utility of a terminal wealth. We are aware of only three papers, Browne (1995), Yang, Zhang (2005) and Wang (2006), in which the expected value of an exponential utility of a terminal reserve is maximized in order to arrive at optimal investment strategies.

The application of a quadratic disutility in the field of a non-life insurance business seems to be quite new. At the same time we must notice that the target-based approach to decision making under uncertainty and the mean square objective is very common in pensions, see for example Gerrard *et al* (2004), Delong *et al* (2007) and references therein. In Delong (2005) a quadratic problem, without a terminal constraint but with a running cost penalizing deviations from a deterministic target, is considered for a general insurer operating in a Black-Scholes market and covering claims generated by a compound Poisson process. Very recently, the Markowitz's portfolio selection problem for a non-life insurer has been solved, and the efficient frontier has been derived in Wang *et al* (2006). We would like to point out that there are significant differences between our paper and their. In Wang *et al* (2006) martingale methods and Backward Stochastic Differential Equations techniques are applied in order to find the optimal investment strategy for an insurer which risk process is modelled as a Lévy process and the surplus is invested in a standard Black-Scholes financial market. The terminal mean-variance objective is only investigated. In this paper, we arrive at the optimal strategy by solving the Hamilton-Jacobi-Bellman equation in the case when an insurer's risk process is modelled as a compound Cox process (a non-Lévy process) with a stochastic, diffusion-type, claim intensity and the surplus is invested in a financial market with an asset which price is driven by a Lévy process (possibly infinite active). We additionally consider a running cost, with a random target process to be reached by the insurer, and this leads to a so-called wealth-path dependent optimization problem, see Bouchard, Pham (2004). Moreover, the strategy derived in Wang *et al* (2006) is expressed in the terms of an "artificial" process, resulting from solving the backward stochastic differential equation, which would be difficult to applied in the real-life setting. Our strategy is expressed directly in the terms of the insurer's wealth process and can easily be applied. As far as the mean-variance optimization objectives are concerned, it is also worth mentioning the paper of Bäuerle (2005), where the optimal proportional reinsurance strategy is derived in a Cramér-Lundberg model.

In the financial literature the mean-variance objective is usually applied to solve portfolio selection problems for self-financing wealth processes. In Zhou, Li (2000) the original mean-variance problem is embedded into an auxiliary problem, which is then solved by the stochastic maximum principle. Techniques of Backward Stochastic Differential Equations are applied in Bielecki *et al* (2005) and Lim (2004), in the presence of random market coefficients. We refer the reader to Guo, Xu (2004) where the Markowitz's portfolio selection is solved in a financial market consisting of assets which prices are driven by a jump-diffusion process, as well as to discrete time model in Cakmak, Özekici (2006), where random returns of assets depend on the state of an observable Markov chain. We would like to point out that in all the above mentioned works there is no running cost included, except in Gerrard *et al* (2004) and Delong (2005), an insurance risk process is modelled as a Brownian motion or as/with a compound Poisson process, except in Wang (2006) and Wang *et al* (2006), where a pure jump process, respectively a Lévy process is considered, a financial market is of Black-Scholes type, except in Guo, Xu (2004), and a target is always deterministic.

This paper is structured as follows. In section 2 we introduce a financial market and an insurance risk process. Our two optimization problems are formulated in section 3 and then solved in section 4. In section 5, in the case of the Markowitz's classical mean-variance portfolio selection for the company, the efficient frontier is derived. Some numerical example is also investigated. We end with summarizing comments in section 6. The proofs are postponed to the appendix.

2 The model

Let us consider a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with a filtration $\mathbb{F} = (\mathcal{F}_t)_{0\le t\le T}$ for some finite T which denotes the investment time horizon. The filtration satisfies the usual hypotheses of completeness ($\mathcal{F}_0$ contains all sets of $\mathbb{P}$-measure zero) and right continuity ($\mathcal{F}_t = \mathcal{F}_{t+}$). The filtration $\mathbb{F}$ consists of three subfiltrations: we set $\mathbb{F} = \mathbb{F}^F \vee \mathbb{F}^C \vee \mathbb{F}^I$, where $\mathbb{F}^F$ contains information about a financial market, $\mathbb{F}^C$ contains information about a claim process and $\mathbb{F}^I$ contains information about a claim intensity. We assume that the subfiltrations $\mathbb{F}^F$ and $(\mathbb{F}^C, \mathbb{F}^I)$ are independent. The measure $\mathbb{P}$ is the real-world, objective probability measure. All expected values are taken with respect to measure $\mathbb{P}$ and the conditional expected value $\mathbb{E}^{\mathbb{P}}[\cdot|X(t) = x, \lambda(t) = \lambda]$ is denoted as $\mathbb{E}^{t,x,\lambda}[\cdot]$. The class of functions $\mathcal{C}^{1,2,2}([0,T)\times\mathbb{R}\times(0,\infty))\cap\mathcal{C}([0,T]\times\mathbb{R}\times(0,\infty))$, which consists of functions continuous on $[0,T]\times\mathbb{R}\times(0,\infty)$ and once continuously differentiable with respect to time variable and twice continuously differentiable with respect to space variables on $[0,T)\times\mathbb{R}\times(0,\infty)$, is denoted simply by $\mathcal{C}$.

In the following subsections we introduce a financial market, a claim process and a claim intensity process.

2.1 The financial market

We consider a Lévy diffusion version of a Black-Scholes financial market. The price of a risk-free asset $S_0 := (S_0(t), 0 \le t \le T)$ is described by the ordinary differential equation

$$\frac{dS_0(t)}{S_0(t)} = rdt, \quad S_0(0) = 1, \tag{2.1}$$

where r denotes a rate of interest. The second tradeable financial instrument in the market is a risky stock and the dynamics of its price $S := (S(t), 0 \le t \le T)$ is given by the stochastic differential equation

$$\frac{dS(t)}{S(t-)} = \mu dt + \xi dL(t), \quad S(0) = 1, \tag{2.2}$$

where μ and ξ denote a drift and a volatility, and $L := (L(t), 0 \le t \le T)$ denotes a zero-mean Lévy process (a process with independent and stationary increments), $\mathbb{F}^F$-adapted with càdlàg sample paths (paths which are continuous on the right and have limits on the left). It is argued that Lévy processes can capture price movements in a much better way, see Cont, Tankov (2004) and Kyprianou *et al* (2005).

The zero-mean Lévy process L is assumed to satisfy the following Lévy-Itô decomposition

$$L(t) = \sigma W(t) + \int_{(0,t]}\int_{\mathbb{R}} z\big(M(ds\times dz) - \nu(dz)ds\big), \tag{2.3}$$

where $W := (W(t), 0 \leq t \leq T)$ is a Brownian motion and $M((s,t] \times A) = \#\{s < u \leq t : (L(u) - L(u-)) \in A\}$ is a Poisson random measure, independent of W, with a deterministic, time-homogeneous intensity measure $\nu(dz)dt$. This Poisson random measure counts the number of jumps of a particular size in a given time interval. Let us recall that $\tilde{M}((0,t] \times A) = M((0,t] \times A) - t\nu(A)$ is a martingale-valued measure, that is $\tilde{M} := (\tilde{M}((0,t] \times A), 0 \leq t \leq T)$ is a $\mathbb{F}^F$-martingale for all Borel sets $A \in \mathcal{B}(\mathbb{R} - \{0\})$. We refer the reader to Applebaum (2004) for mathematical details concerning Lévy processes and Poisson random measures.

We make the following assumptions concerning the coefficients and the intensity measure:

(A1) r, μ, σ are non-negative constants and $r < \mu$,

(A2) we set $\xi \equiv 1$, this is no loss of generality as the process ξL has also independent and stationary increments and satisfies the Lévy-Itô decomposition,

(A3) ν is a Lévy measure on $(-1, \infty)$, such that $\nu(\{0\}) = 0$ and $\int_{z \geq 1} z^4 \nu(dz) < \infty$.

We recall that a Lévy measure is a measure which verifies $\int_{\mathbb{R}} (z^2 \wedge 1)\nu(dz) < \infty$. The moment condition in (A3) ensures that $\sup_{t \in [0,T]} \mathbb{E}[|L(t)|^4] < \infty$.

The stochastic differential equation (2.2) has a unique, positive and almost surely finite solution, given explicitly by Doléans-Dade exponential

$$
\begin{aligned}
S(t) &= \exp\big\{\big(\mu - \frac{1}{2}\sigma^2 + \int_{z>-1} (\log(1+z) - z)\nu(dz)\big)t \\
&\quad + \sigma W(t) + \int_{(0,t]} \int_{z>-1} \log(1+z)\tilde{M}(ds \times dz)\big\} \\
&= \exp\big\{\mu_E t + \sigma W(t) + \int_{(0,t]} \int_{\mathbb{R}} z\tilde{M}_E(ds \times dz)\big\} \qquad (2.4)
\end{aligned}
$$

which is an exponential Lévy process with the measure $\nu_E(A) = \nu(\{z : \log(1+z) \in A\})$, see Cont, Tankov (2005), propositions 8.21 and 8.22. The measure ν_E satisfies the equivalent condition (A3'):

(A3') ν_E is a Lévy measure on $\mathbb{R}$, such that $\nu_E(\{0\}) = 0$ and $\int_{z \geq 1} e^{4z} \nu_E(dz) < \infty$.

It is well known that there is one to one correspondence between the measures and the stock price models (2.2) and (2.4). We would like to point out that in chapter 3 in Kyprianou *et al* (2005) the intensity measures ν_E for Variance Gamma and CGMY processes are estimated for five world index markets and in each case the estimated measure satisfies (A3'). We refer the reader to Cont, Tankov (2005) and Kyprianou *et al* (2005) in which different aspects of financial modelling with Lévy diffusion processes are investigated.

2.2 The insurance risk process

We consider a collective insurance risk model. Let $C(t)$ denote an aggregate claim amount paid up to time t. We assume that the process $C := (C(t), 0 \leq t \leq T)$ is a compound Cox process which means that it has the following representation

$$C(t) = \sum_{i=1}^{N(t)} Y_i, \tag{2.5}$$

where $\{Y_i, i \in \mathbb{N}\}$ is a sequence of positive, independent and identically distributed random variables with a distribution $F(y) = \mathbb{P}(Y_i \leq y)$ and $N := (N(t), 0 \leq t \leq T)$ is a Cox process with a stochastic intensity process $\Lambda := (\lambda(t), 0 \leq t \leq T)$ and a law given by

$$\mathbb{P}(N(t) = k|\mathcal{F}_t^I) = \frac{e^{-H(t)}(H(t))^k}{k!}, \quad k = 0, 1, 2..., \tag{2.6}$$

where H denotes a cumulative hazard function

$$H(t) = \int_0^t \lambda(s)ds. \tag{2.7}$$

We assume that the aggregate claim process C is $\mathbb{F}^C$-adapted with càdlàg sample paths. Clearly, $Y_1, Y_2, ...$ denote the amounts of successive claims and N counts the number of claims.

A Cox process is a common alternative to a Poisson process in the risk theory and in insurance modelling of claim processes, see Klüppelberg, Mikosch (1995), Rolski *et al* (1999), Dassios, Jang (2006) and references therein. Notice that a Cox process does not have independent and stationary increments and is of finite variation on $[0, T]$.

We would like to point out that the compound Cox process can also be defined through a measure $N(dt, dz)$ as

$$C(t) = \int_{(0,t]} \int_0^\infty zN(ds \times dz), \tag{2.8}$$

where $N((s,t] \times A) = \#\{s < u \leq t : (C(u) - C(u-)) \in A\}$ is a finite random measure with a random compensator $dF(z)\lambda(t)dt$. We find it convenient to use the representation (2.8) in the proof of the verification theorem.

We assume that the dynamics of the claim intensity Λ is given by the stochastic differential equation

$$d\lambda(t) = \theta(t, \lambda(t))dt + \eta(t, \lambda(t))d\bar{W}(t), \quad \lambda(0) = \lambda, \tag{2.9}$$

where $\bar{W} := (\bar{W}(t), 0 \leq t \leq T)$ is an $\mathbb{F}^I$-adapted Brownian motion, independent of W and $\tilde{M}$.

We make the following assumptions concerning the insurance risk process:

(B1) the distribution F has a finite fourth moment, $\int_0^\infty z^4 dF(z) < \infty$,

(B2) $\theta : [0,T] \times (0,\infty) \to \mathbb{R}, \eta : [0,T] \times (0,\infty) \to (0,\infty)$ are continuous functions, locally Lipschitz continuous in λ, uniformly in t,

(B3) there exists a sequence $(E_n)_{n\in\mathbb{N}}$ of bounded domains with $\bar{E}_n \subseteq (0,\infty)$ and $\bigcup_{n\geq 1} E_n = (0,\infty)$, such that the functions $\theta(t,\lambda)$ and $\eta(t,\lambda)$ are uniformly Lipschitz continuous on $[0,T] \times \bar{E}_n$,

(B4) $\mathbb{P}(\forall_{s\in[t,T]}\lambda(s) \in (0,\infty)|\lambda(t) = \lambda) = 1$ for all starting points $(t,\lambda) \in [0,T] \times (0,\infty)$,

(B5) $\sup_{s\in[t,T]} \mathbb{E}^{t,\lambda}[|\lambda(s)|^4] < \infty$ for all starting points $(t,\lambda) \in [0,T] \times (0,\infty)$.

Under the assumptions (B2) and (B4), for each starting point $(t,\lambda) \in [0,T]\times(0,\infty)$, the intensity process is nonexplosive on $[t,T]$ and there exists a unique strong solution to the stochastic differential equation (2.9), such that the mapping $(t,\lambda,s) \to \lambda^{t,\lambda}(s)$ is $\mathbb{P}$-a.s. continuous, see Heath, Schweizer (2000), Becherer, Schweizer (2005). The assumptions (B1), (B3), (B5) together with (A3)/(A3'), are required in the verification result and ensure that a classical solution to our Hamilton-Jacobi-Bellman equation exists.

We remark that from the point of the company the assumption concerning the measurability of the claim intensity and its diffusion type is very reasonable. In the literature one can find that, for example, a discontinuous shot noise process is quite useful in modelling claim intensities, see Klüppelberg, Mikosch (1995), Dassios, Jang (2005) for comments. However, it is proved in Dassios, Jang (2005), and intuitively it is clear, that a shot noise process converges to a diffusion process in the case of high frequency events, such as accidents from a large collective portfolio. Moreover, the company should also have enough data to estimate the intensity correctly.

3 Problem formulation

In this paper we investigate two optimization problems which are quite similar in mathematical formulation but the motivation for stating and solving them is a little different.

Firstly, let us deal with a portfolio selection for a general insurance company. Let us consider a wealth process of the insurer $X^\pi := (X^\pi(t), 0 \leq t \leq T)$. Its dynamics are given by the stochastic differential equation

$$\begin{aligned} dX^\pi(t) &= \pi(t)\big(\mu dt + \sigma dW(t) + \int_{z>-1} z\tilde{M}(dt \times dz)\big) \\ &\quad + \big(X^\pi(t-) - \pi(t)\big)rdt + c(t)dt - dC(t), \quad X(0) = x, \end{aligned} \tag{3.1}$$

where π denotes the amount of the wealth invested in the risky asset, c denotes a premium rate and x denotes an initial capital. We assume, as in the classical risk theory, see Rolski *et al* (1999), that

$$c(t)dt = (1+\theta)\mu_1\lambda(t-)dt, \quad \mu_1 = \int_0^\infty z dF(z), \tag{3.2}$$

which means that the premium collected over an infinitesimal interval dt is equal to the expected value of claims paid over this interval with an additional safety loading θ. According to the classical portfolio theory due to Markowitz an investment strategy should be chosen in the following way

$$\begin{cases} \inf_\pi \mathrm{Var}[X^\pi(T)] \\ \mathbb{E}[X^\pi(T)] = P \end{cases}, \tag{3.3}$$

where P is a specified target.

The second optimization problem which we consider, can be applied to an individual policy. In this case the value process of the issued policy X^π evolves according to the stochastic differential equation

$$\begin{aligned} dX^\pi(t) &= \pi(t)\big(\mu dt + \sigma dW(t) + \int_{z>-1} z\tilde{M}(dt\times dz)\big) \\ &\quad + \big(X^\pi(t-) - \pi(t)\big)rdt - dC(t), \quad X(0) = x, \end{aligned} \tag{3.4}$$

where, this time, x denotes a single premium for the contract. The optimization functional can include, apart from the terminal mean-variance objective, a running cost penalizing deviations of the value process from a target process $(\bar{R}(t), 0 \le t \le T)$. An investment strategy can be chosen in the following way

$$\begin{cases} \inf_\pi \mathbb{E}\big[\int_0^T \big(X^\pi(t) - \bar{R}(t)\big)^2 dt\big] + \alpha \mathrm{Var}[X^\pi(T) - \bar{R}(T)] \\ \mathbb{E}\big[X^\pi(T) - \bar{R}(T)\big] = 0 \end{cases}, \tag{3.5}$$

where $\alpha > 0$ attaches a weight to the terminal cost. The target can combine two elements

$$\bar{R}(t) = P(t) + R(t, \lambda(t)), \quad t \in [0, T] \tag{3.6}$$

where $P(t)$ denotes a profit which should accumulate until time t, for example $P(t) = xe^{\rho t}, \rho > 0$, and $R(t, \lambda)$ denotes a reserve which should be kept at time t, when the claim intensity equals λ, in order to meet future contractual obligations. We need the following technical assumption concerning the profit function P:

(C) $P : [0, T] \to [0, \infty)$ is Lipschitz continuous.

We believe that the inclusion of the running cost in the optimization problem is very reasonable as it means that the optimally controlled policy should generate values satisfying the desired profit as well as the required solvency constraints during the whole term of the contract.

We assume that the reserve for outstanding liabilities R is calculated as the expected present value of future undiscounted payments, conditioned on the given level of the claim intensity. It is straightforward to calculate the value of the reserve

$$R(t,\lambda) = \mu_1 \mathbb{E}^{t,\lambda}\Big[\int_t^T \lambda(s)ds\Big]. \tag{3.7}$$

We can prove the following lemma.

Lemma 3.1. *The function $R : [0,T] \times (0,\infty) \to [0,\infty)$, defined in (3.7), is in the class $\mathcal{C}$ and satisfies the following partial differential equation*

$$\frac{\partial R}{\partial t}(t,\lambda) + \theta(t,\lambda)\frac{\partial R}{\partial \lambda}(t,\lambda) + \frac{1}{2}\eta^2(t,\lambda)\frac{\partial^2 R}{\partial \lambda^2}(t,\lambda) + \mu_1\lambda = 0, R(T,\lambda) = 0. \tag{3.8}$$

Moreover, the equation (3.8) has the unique solution in the class $\mathcal{C}$.

4 Solution of optimization problems

In this part of the paper we solve both problems stated in the previous section. We deal with the following constrained optimization problem

$$\begin{cases} \inf_\pi \alpha \mathbb{E}\big[\int_0^T \big(X^\pi(t) - \bar{R}(t)\big)^2 dt\big] + (1-\alpha)\mathrm{Var}[X^\pi(T) - \bar{R}(T)] \\ \mathbb{E}\big[X^\pi(T) - \bar{R}(T)\big] = 0 \end{cases}, \tag{4.1}$$

where $\alpha \in [0,1)$ and the dynamics of the wealth process X^π is given by the stochastic differential equation

$$\begin{aligned} dX^\pi(t) &= \pi(t)\big(\mu dt + \sigma dW(t) + \int_{z>-1} z\tilde{M}(dt \times dz)\big) \\ &\quad + \big(X^\pi(t-) - \pi(t)\big)r dt + (1+\theta)\mu_1\lambda(t)dt - dC(t), \quad X(0) = x, \end{aligned} \tag{4.2}$$

Clearly, by setting $\alpha = 0$ and $\bar{R}(T) = P$ we recover the optimization problem (3.3), while setting $\theta = -1$ leads to the optimization problem (3.5).

The variance criterion can be handled by incorporating the equality constraint on the expected value into the objective function by using a Lagrange multiplier. First we can solve the stochastic control problem

$$\begin{aligned} \inf_{\pi \in \mathcal{A}} \mathbb{E}^{0,x,\lambda}\big[\alpha \int_0^T \big(X^\pi(t) - P(t) - R(t,\lambda(t))\big)^2 dt \\ +(1-\alpha)\big(X^\pi(T) - P(T)\big)^2 - \beta\big(X^\pi(T) - P(T)\big)\big], \end{aligned} \tag{4.3}$$

and then choose a Lagrange multiplier β such that the constraint on the expected value of the terminal wealth is satisfied

$$\mathbb{E}^{0,x,\lambda}[X^{\hat{\pi},\beta}(T)] = P(T), \tag{4.4}$$

where $\hat{\pi}$ is the optimal strategy for (4.3).

Let us introduce the set of admissible strategies and three operators.

Definition 4.1. *A strategy* $(\pi(t), 0 < t \le T)$ *is admissible,* $\pi \in \mathcal{A}$*, if it satisfies the following conditions:*

1. $\pi : (0,T] \times \Omega \to \mathbb{R}$ *is a predictable mapping with respect to filtration* $\mathbb{F}$,
2. $\mathbb{E}^{0,x,\lambda}[\int_0^T \pi^2(t)dt] < \infty$,
3. *the stochastic differential equation (4.2) has a unique solution* X^π *on* $[0,T]$.

It is well-known that it is sufficient to consider only Markov strategies, see Øksendal, Sulem (2005), chapter 3. We point out that for any $\pi \in \mathcal{A}$ the process X^π, which satisfies (4.2), is a square integrable semimartingale with cádlág sample paths, see Applebaum (2004), chapter 4.3.3. Moreover, for all $t \in [0,T]$ the following inequality holds

$$|R(t,\lambda(t))|^2 \le T\mu_1^2\mathbb{E}[\int_t^T \lambda^2(s)ds|\mathcal{F}_t], \quad \mathbb{P}-a.s., \tag{4.5}$$

due to the Markov property of the claim intensity process, Jensen's inequality for conditional expectations and the Cauchy-Schwartz inequality. The estimate (4.5) allows us to prove the square integrability of the reserve process $(R(t,\lambda(t)), 0 \le t \le T)$. For all $t \in [0,T]$ we have

$$\mathbb{E}^{0,\lambda}[|R(t,\lambda(t))|^2] \le T\mu_1^2\mathbb{E}^{0,\lambda}[\int_t^T \lambda^2(s)ds] \le T\mu_1^2\mathbb{E}^{0,\lambda}[\int_0^T \lambda^2(s)ds] < \infty, \tag{4.6}$$

where the law of iterated expectations has been applied. We can conclude that the objective function (4.3) is well-defined.

Definition 4.2. *The integro-differential operator* $\mathcal{L}_F$ *is given by*

$$\begin{aligned}\mathcal{L}_F^\pi\phi(t,x,\lambda) &= \big(\pi(\mu-r) + xr + (1+\theta)\mu_1\lambda\big)\frac{\partial\phi}{\partial x}(t,x,\lambda) + \frac{1}{2}\pi^2\sigma^2\frac{\partial^2\phi}{\partial x^2}(t,x,\lambda)\\ &\quad + \int_{z>-1}\big(\phi(t,x+\pi z,\lambda) - \phi(t,x,\lambda) - \pi z\frac{\partial\phi}{\partial x}(t,x,\lambda)\big)\nu(dz), \end{aligned}\tag{4.7}$$

the integral operator $\mathcal{L}_C$ *is given by*

$$\mathcal{L}_C\phi(t,x,\lambda) = \lambda\int_0^\infty \big(\phi(t,x-z,\lambda) - \phi(t,x,\lambda)\big)dF(z), \tag{4.8}$$

and the differential operator $\mathcal{L}_I$ *is given by*

$$\mathcal{L}_I\phi(t,x,\lambda) = \theta(t,\lambda)\frac{\partial\phi}{\partial\lambda}(t,x,\lambda) + \frac{1}{2}\eta^2(t,\lambda)\frac{\partial^2\phi}{\partial\lambda^2}(t,x,\lambda). \tag{4.9}$$

This operators are defined for all functions ϕ *such that the partial derivatives and the integrals in (4.7), (4.8) and (4.9) exist pointwise.*

Let us introduce the optimal value function for the problem (4.3)

$$\begin{aligned} V(t,x,\lambda) &= \inf_{\pi\in\mathcal{A}} \mathbb{E}^{t,x,\lambda}\big[\alpha\int_0^T \big(X^\pi(t)-P(t)-R(t,\lambda(t))\big)^2 dt \\ &\quad +(1-\alpha)\big(X^\pi(T)-P(T)\big)^2-\beta\big(X^\pi(T)-P(T)\big)\big], \end{aligned} \tag{4.10}$$

In the appendix we prove the following classical verification theorem.

Theorem 4.1. *Let $v\in\mathcal{C}$ satisfy for all $\pi\in\mathcal{A}$*

$$\begin{aligned} 0 &\le \alpha\big(x-P(t)-R(t,\lambda)\big)^2+\frac{\partial v}{\partial t}(t,x,\lambda)+\mathcal{L}_F^\pi v(t,x,\lambda) \\ &\quad +\mathcal{L}_I v(t,x,\lambda)+\mathcal{L}_C v(t,x,\lambda), \end{aligned} \tag{4.11}$$

for all $(t,x,\lambda)\in[0,T)\times\mathbb{R}\times(0,\infty)$, with

$$v(T,x,\lambda)=(1-\alpha)\big(x-P(T)\big)^2-\beta\big(x-P(T)\big), \forall(x,\lambda)\in\mathbb{R}\times(0,\infty). \tag{4.12}$$

Assume that for all $\pi\in\mathcal{A}$

$$\mathbb{E}^{0,x,\lambda}\big[\int_0^T\int_{z>-1}|v(t,X^\pi(t-)+\pi(t)z,\lambda(t))-v(t,X^\pi(t-),\lambda(t))|^2\nu(dz)dt\big]<\infty, \tag{4.13}$$

$$\begin{aligned} &\mathbb{E}^{0,x,\lambda}\big[\int_0^T\int_{z>-1}|v(t,X^\pi(t-)+\pi(t)z,\lambda(t))-v(t,X^\pi(t-),\lambda(t)) \\ &-\pi(t)z\frac{\partial v}{\partial x}(t,X^\pi(t-),\lambda(t))|\nu(dz)dt\big]<\infty, \end{aligned} \tag{4.14}$$

$$\mathbb{E}^{0,x,\lambda}\big[\int_0^T\int_0^\infty|v(t,X^\pi(t-)-z,\lambda(t))-v(t,X^\pi(t),\lambda(t))|^2\lambda(t)dF(z)dt\big]<\infty, \tag{4.15}$$

and

$\{v^+(\tau,X^\pi(\tau),\lambda(\tau))\}_{0<\tau\le T}$ *is uniformly integrable for all $\mathbb{F}$-stopping times τ.* (4.16)

Then

$$v(t,x,\lambda)\le V(t,x,\lambda),\quad \forall(t,x,\lambda)\in[0,T]\times\mathbb{R}\times(0,\infty). \tag{4.17}$$

Moreover, if there exists an admissible control $\hat\pi$ such that

$$\begin{aligned} 0 &= \alpha\big(x-P(t)-R(t,\lambda)\big)^2+\frac{\partial v}{\partial t}(t,x,\lambda)+\mathcal{L}_F^{\hat\pi} v(t,x,\lambda) \\ &\quad +\mathcal{L}_I v(t,x,\lambda)+\mathcal{L}_C v(t,x,\lambda), \end{aligned} \tag{4.18}$$

for all $(t,x,\lambda)\in[0,T)\times\mathbb{R}\times(0,\infty)$, and

$\{v(\tau,X^{\hat\pi}(\tau),\lambda(\tau))\}_{0<\tau\le T}$ *is uniformly integrable for all $\mathbb{F}$-stopping times τ,* (4.19)

then

$$v(t,x,\lambda)=V(t,x,\lambda),\quad \forall(t,x,\lambda)\in[0,T]\times\mathbb{R}\times(0,\infty). \tag{4.20}$$

and $\hat\pi$ is the optimal strategy.

In the above theorem $v^+ := \max(v, 0)$ denotes a positive part of a function v.

As our optimization problem (4.3) is quadratic it is natural to try to find a solution in the form $v(t,x,\lambda) = A(t,\lambda)x^2 + B(t,\lambda)x + C(t,\lambda)$. With this choice of the value function the optimal strategy $\hat{\pi}$ which minimizes the right hand side of (4.11) is given by

$$\hat{\pi}(t,x,\lambda) = -\bar{\delta}\left(x + \frac{B(t,\lambda)}{2A(t,\lambda)}\right), \quad \bar{\delta} = \frac{\mu - r}{\sigma^2 + \int_{z>-1} z^2 \nu(dz)}. \tag{4.21}$$

Substituting (4.21) into (4.18) and collecting the terms we arrive at three partial differential equations

$$\begin{cases} \frac{\partial A}{\partial t}(t,\lambda) + \mathcal{L}_I A(t,\lambda) + (2r-\delta)A(t,\lambda) + \alpha = 0, \\ A(T,\lambda) = 1-\alpha, \end{cases} \tag{4.22}$$

$$\begin{cases} \frac{\partial B}{\partial t}(t,\lambda) + \mathcal{L}_I B(t,\lambda) + (r-\delta)B(t,\lambda) - 2\alpha\big(P(t) + R(t,\lambda)\big) \\ +2\theta\mu_1\lambda A(t,\lambda) = 0, \\ B(T,\lambda) = -2(1-\alpha)P(T) - \beta, \end{cases} \tag{4.23}$$

$$\begin{cases} \frac{\partial C}{\partial t}(t,\lambda) + \mathcal{L}_I C(t,\lambda) + \lambda\mu_2 A(t,\lambda) + \theta\mu_1\lambda B(t,\lambda) + \alpha\big(P(t) + R(t,\lambda(t))\big)^2 \\ -\frac{B^2(t,\lambda)}{4A(t,\lambda)}\delta = 0, \\ C(T,\lambda) = (1-\alpha)P^2(T) + \beta P(T), \end{cases} \tag{4.24}$$

where $\delta = \bar{\delta}(\mu - r)$ and $\mu_2 = \int_0^\infty z^2 dF(z)$.

A solution to (4.22) can be stated explicitly by noticing that the time-dependent function of the form

$$A(t) = (1-\alpha)e^{(2r-\delta)(T-t)} + \frac{\alpha}{2r-\delta}(e^{(2r-\delta)(T-t)} - 1) \tag{4.25}$$

satisfies (4.22). It is easy to check that the function $A(t)$ is strictly positive and its inverse is bounded uniformly on $[0,T]$. As far as the next two partial differential equations are concerned we can prove the following lemma.

Lemma 4.1. *There exist unique solutions $B, C \in \mathcal{C}$ to the partial differential equations (4.23) and (4.24). These solutions satisfy*

$$|B(t,\lambda)| \leq K\big(1 + \mathbb{E}^{t,\lambda}\big[\int_t^T \lambda(s)ds\big]\big), \tag{4.26}$$

$$|C(t,\lambda)| \leq K\big(1 + \mathbb{E}^{t,\lambda}\big[\int_t^T \lambda^2(s)ds\big]\big), \tag{4.27}$$

for all $(t,\lambda) \in [0,T] \times (0,\infty)$ and some finite constant $K > 0$. In particular, one can state the Feynman-Kac representation of the unique solution to (4.23)

$$\begin{aligned} B(t,\lambda) &= -\big(2(1-\alpha)P(T) + \beta\big)e^{(r-\delta)(T-t)} - 2\alpha\int_t^T P(s)e^{(r-\delta)(s-t)}ds \\ &\quad +2\mu_1\mathbb{E}^{t,\lambda}\Big[\int_t^T \left(\theta A(s)e^{(r-\delta)(s-t)} - \alpha\frac{e^{(r-\delta)(s-t)} - 1}{r-\delta}\right)\lambda(s)ds\Big]. \end{aligned} \tag{4.28}$$

Notice that after specifying the claim intensity process and calculating its expected value, the solution to (4.23) might be given explicitly through (4.28), after integrating some deterministic functions.

Let us now investigate the wealth process $X^{\hat{\pi}}$ under the optimal strategy. Its dynamics is given by the stochastic differential equation

$$\begin{aligned} dX^{\hat{\pi}}(t) &= \big\{ -\delta\left(X^{\hat{\pi}}(t-)+\frac{B(t,\lambda(t))}{2A(t)}\right)+X^{\hat{\pi}}(t-)r+(1+\theta)\mu_1\lambda(t)\big\}dt-dC(t) \\ &\quad -\bar{\delta}\left(X^{\hat{\pi}}(t-)+\frac{B(t,\lambda(t))}{2A(t)}\right)\left(\sigma dW(t)+\int_{z>-1} z\tilde{M}(dt\times dz)\right), \end{aligned} \tag{4.29}$$

with the initial condition $X(0)=x$. We can arrive at the following result.

Lemma 4.2. *The stochastic differential equation (4.29), given the initial condition $X(0)=x\in\mathbb{R}$, has a unique solution on $[0,T]$ in the space of semimartingale processes with càdlàg sample paths. This solution has a finite fourth moment, $\sup_{t\in[0,T]}\mathbb{E}^{0,x,\lambda}[|X^{\hat{\pi}}(t)|^4]<\infty$.*

We proceed now to find the value of the Lagrange multiplier. The Itô differential (4.29) can be rewritten in the integral form

$$\begin{aligned} X^{\hat{\pi}}(t) &= x+\int_0^t \big\{ -\delta\left(X^{\hat{\pi}}(s-)+\frac{B(s,\lambda(s))}{2A(s)}\right) \\ &\quad +X^{\hat{\pi}}(s-)r+(1+\theta)\mu_1\lambda(s)\big\}ds-C(t) \\ &\quad -\int_0^t \bar{\delta}\left(X^{\hat{\pi}}(s-)+\frac{B(s,\lambda(s))}{2A(s)}\right)\left(\sigma dW(s)+\int_{z>-1} z\tilde{M}(ds\times dz)\right) \end{aligned} \tag{4.30}$$

Taking the expected value on both sides of (4.30) and applying Fubini's theorem we arrive at

$$\varphi(t)=x+\int_0^t \{-\delta(\varphi(s-)+\frac{B(s)}{2A(s)})+r\varphi(s-)+\theta\mu_1 m(s)\}ds, \tag{4.31}$$

where we define for $0\le s\le T$

$$\varphi(s) = \mathbb{E}^{0,x,\lambda}[X^{\hat{\pi}}(s)], \tag{4.32}$$

$$m(s) = \mathbb{E}^{0,\lambda}[\lambda(s)], \tag{4.33}$$

and

$$\begin{aligned} B(s) &= -\big(2(1-\alpha)P(T)+\beta\big)e^{(r-\delta)(T-s)}-2\alpha\int_s^T P(u)e^{(r-\delta)(u-s)}du \\ &\quad +2\mu_1\int_s^T\left(\theta A(u)e^{(r-\delta)(u-s)}-\alpha\frac{e^{(r-\delta)(u-s)}-1}{r-\delta}\right)m(u)du. \end{aligned} \tag{4.34}$$

We point out that the expected values of the stochastic integrals in (4.30) are indeed equal to zero due to the square integrability of the processes $X^{\hat{\pi}}$ and $(B(t,\lambda(t)),0\le$

$t \leq T$). In the view of lemma 4.2 the first statement is clear, and in order to establish the second one, notice that for all $t \in [0,T]$ the following inequalities hold $\mathbb{P}$-a.s.

$$|B(t,\lambda(t))|^2 \leq K\big(1+\mathbb{E}[\int_t^T \lambda(s)ds|\mathcal{F}_t]\big)^2 \leq K_1\big(1+\mathbb{E}[\int_t^T \lambda^2(s)ds|\mathcal{F}_t]\big), \quad (4.35)$$

due to the Markov property of the intensity process, the estimate (4.26), Jensen's inequality for conditional expectations and the Cauchy-Schwartz inequality. Then, taking the expected value on both sides of (4.35) and applying the law of iterated expectations we arrive at

$$\begin{aligned}\mathbb{E}^{0,\lambda}[|B(t,\lambda(t)|^2] &\leq K_1\big(1+\mathbb{E}^{0,\lambda}[\int_t^T \lambda^2(s)ds]\big)\\ &\leq K_1\big(1+\mathbb{E}^{0,\lambda}[\int_0^T \lambda^2(s)ds]\big) < \infty, \end{aligned} \quad (4.36)$$

for all $t \in [0,T]$. The function (4.34) arises due to the Markov property of the intensity process and the law of iterated expectations.

It is easy to show that the function φ satisfying (4.31) must be continuous and differentiable. The integral equation (4.31) can be transformed back into the ordinary differential equation

$$\frac{d\varphi}{dt}(t) = \big(r-\delta\big)\varphi(t) + \theta\mu_1 m(t) - \delta\frac{B(t)}{2A(t)}, \quad \varphi(0) = x, \quad (4.37)$$

which can be solved resulting in

$$\varphi(T) = xe^{(r-\delta)T} + \theta\mu_1\int_0^T m(t)e^{(r-\delta)(T-t)}dt - \delta\int_0^T \frac{B(t)}{2A(t)}e^{(r-\delta)(T-t)}dt. \quad (4.38)$$

It is left to find the value of β such that the constraint $\varphi(T) = P(T)$ is satisfied. With a little algebra we can arrive at the value of the Lagrange multiplier

$$\beta = 2\frac{\big(1-\delta(1-\alpha)\beta_2\big)P(T) - xe^{(r-\delta)T} - \beta_1 - \delta\beta_3}{\delta\beta_2}, \quad (4.39)$$

where

$$\beta_1 = \theta\mu_1\int_0^T m(t)e^{(r-\delta)(T-t)}dt, \quad (4.40)$$

$$\beta_2 = \int_0^T \frac{e^{2(r-\delta)(T-t)}}{A(t)}dt, \quad (4.41)$$

and

$$\begin{aligned}\beta_3 &= \mu_1\int_0^T \frac{e^{(r-\delta)(T-t)}}{A(t)}\int_t^T \Big(\alpha\frac{e^{(r-\delta)(s-t)}-1}{r-\delta} - \theta A(s)e^{(r-\delta)(s-t)}\Big)m(s)dsdt\\ &\quad +\alpha\int_0^T \frac{e^{(r-\delta)(T-t)}}{A(t)}\int_t^T P(s)e^{(r-\delta)(s-t)}dsdt. \end{aligned} \quad (4.42)$$

We conclude with the theorem summarizing our results.

Theorem 4.2. *The investment strategy given by*

$$\hat{\pi}(t) = -\bar{\delta}\left(X^{\hat{\pi}}(t-) + \frac{B(t,\lambda(t))}{2A(t)}\right), \quad \bar{\delta} = \frac{\mu - r}{\sigma^2 + \int_{z>-1} z^2\nu(dz)}, \tag{4.43}$$

is the optimal investment strategy for the constrained quadratic optimization problem (4.1), and the minimum cost is equal to $A(0)x^2 + B(0,\lambda)x + C(0,\lambda)$*. The functions* A*,* B*,* C *and the constant* β *are given by (4.22)-(4.24), (4,39), whereas* $X^{\hat{\pi}}$ *is the wealth process under the optimal strategy, evolving according to (4.29).*

5 Efficient frontier and efficient portfolios

In this section we derive the efficient frontier and the efficient portfolios of the mean-variance portfolio selection problem (3.3) for the general insurance company. Let us recall that an efficient portfolio is one for which there does not exist another portfolio which has higher mean and no higher variance, and/or has less variance and no less mean, see Bielecki *et al* (2005).

As stated in Bielecki *et al* (2005), the efficient frontier can be derived from the variance minimizing frontier. In our case the variance minimizing frontier is just the optimal value function $v(0,x,\lambda)$, which can be recovered from the results of the previous section by setting $\alpha = 0$. After tedious calculations, which we omit, one can show that the Lagrange multiplier is equal to

$$\beta = 2\frac{P - e^{rT}(x + \theta\mu_1\mathbb{E}^{0,\lambda}[\int_0^T e^{-rt}\lambda(t)dt])}{e^{\delta T} - 1}, \tag{5.1}$$

and the variance minimizing frontier, as the function of $P = \mathbb{E}[X^{\hat{\pi}}(T)]$, is equal to

$$\begin{aligned}\mathrm{Var}[X^{\hat{\pi}}(T)] &= \mu_2 e^{(2r-\delta)T}\mathbb{E}^{0,\lambda}\big[\int_0^T e^{-(2r-\delta)t}\lambda(t)dt\big] + \theta^2\mu_1^2 e^{(2r-\delta)T}W \\ &\quad + \frac{\Big(P - e^{rT}\big(x + \theta\mu_1\mathbb{E}^{0,\lambda}[\int_0^T e^{-rt}\lambda(t)dt]\big)\Big)^2}{e^{\delta T} - 1},\end{aligned} \tag{5.2}$$

where

$$\begin{aligned}W &= 2\mathbb{E}^{0,\lambda}\left[\int_0^T e^{\delta t}e^{-rt}\lambda(t)\int_t^T e^{-rs}\lambda(s)dsdt\right] \\ &\quad -\mathbb{E}^{0,\lambda}\left[\int_0^T \delta e^{\delta t}\left(\mathbb{E}\big[\int_t^T e^{-rs}\lambda(s)ds|\mathcal{F}_t\big]\right)^2 dt\right] \\ &\quad -\left(\mathbb{E}^{0,\lambda}\big[\int_0^T e^{-rt}\lambda(t)dt\big]\right)^2.\end{aligned} \tag{5.3}$$

Let us deal with the constant W. Due to Jensen's inequality for conditional expectations, Fubini's theorem and the law of iterated expectations, which are applied to the second term in (5.2), we can arrive at

$$\begin{aligned} W &\geq 2\mathbb{E}^{0,\lambda}\left[\int_0^T e^{\delta t}e^{-rt}\lambda(t)\int_t^T e^{-rs}\lambda(s)dsdt\right] \\ &\quad -\mathbb{E}^{0,\lambda}\left[\int_0^T \delta e^{\delta t}\left(\int_t^T e^{-rs}\lambda(s)ds\right)^2 dt\right] \\ &\quad -\left(\mathbb{E}^{0,\lambda}\Big[\int_0^T e^{-rt}\lambda(t)dt\Big]\right)^2. \end{aligned} \tag{5.4}$$

Let us consider for a moment a continuous function f. It is easy to show, by integrating by parts, that the following equality holds

$$\begin{aligned} 2\int_0^T e^{\delta t}f(t)\int_t^T f(s)dsdt &= \int_0^T e^{\delta t}\frac{d}{dt}\Big(-\int_t^T f(s)ds\Big)^2 dt \\ &= \Big(\int_0^T f(s)ds\Big)^2 + \int_0^T \delta e^{\delta t}\Big(\int_t^T f(s)ds\Big)^2 ds. \end{aligned} \tag{5.5}$$

Let us now put $f(s) = e^{-rs}\lambda(s)$ and apply (5.5) to the first two terms in (5.4) under the expected value. We can arrive at

$$W \geq \mathbb{E}^{0,\lambda}\Big[\Big(\int_0^T e^{-rt}\lambda(t)dt\Big)^2\Big] - \left(\mathbb{E}^{0,\lambda}\Big[\int_0^T e^{-rt}\lambda(t)dt\Big]\right)^2 \geq 0. \tag{5.6}$$

This proves a rather obvious fact that there exists no "risk-free asset" in our economy. Even by choosing the expected return target $P = e^{rT}\big(x + \theta\mu_1\mathbb{E}^{0,\lambda}[\int_0^T e^{-rt}\lambda(t)dt]\big)$, which corresponds to investing at time $t = 0$ all our wealth in the bank account, we are still left with strictly positive variance of the surplus due to possible claims.

We can state the following straightforward lemma.

Lemma 5.1. *Let $P^* = e^{rT}\big(x + \theta\mu_1\mathbb{E}^{0,\lambda}[\int_0^T e^{-rt}\lambda(t)dt]\big)$. The variance minimizing frontier (5.1) is strictly decreasing for $(-\infty, P^*)$ and strictly increasing for (P^*, ∞). The efficient frontier is the subset of the variance minimizing frontier corresponding to $P \in [P^*, \infty)$.*

It is intuitively clear, that the insurer should set the target such that $P \geq P^*$ holds, as it should require the profit which would be not less than the expected value of the future cash inflows/outflows invested in the bank account.

The efficient portfolios are those portfolios whose expected returns and corresponding variances lie on the efficient frontier. We point out that the capital market line, drawn in a mean-standard deviation plane, is no longer a straight line. Finally, let us recall the well-known facts, see Bielecki *et al* (2005), that in the case without claims, the variance of the terminal wealth could be represented as a complete square, there would be a "risk free asset" - a bank account, and the capital market line would be a straight line.

Example 5.1. Let us assume that the stock price follows an exponential Variance Gamma process of the form

$$S(t) = e^{0,28t+L(t)}, \quad L(t) = -0,2h(t) + 0,2W(h(t)), \tag{5.7}$$

where $h(t)$ is a Gamma distributed random variable with the density function

$$g_{h(t)}(y) = \frac{1}{\Gamma(t/0,003)(0,003)^{t/0,003}} y^{\frac{t}{0,003}-1} e^{-\frac{y}{0,003}}. \tag{5.8}$$

For the subordinated Brownian motion representation of a Variance Gamma process we refer the reader to Cont, Tankov (2004) or Kyprianou *et al* (2005). We remark that this choice of parameters corresponds to $\mu = 0,1$. We assume that the individual claims are exponentially distributed, with the expected value equals to $\mu_1 = 10$, and that the claim intensity process follows an exponential martingale of the form

$$\lambda(t) = 100e^{0,2\bar{W}(t)-0,02t}. \tag{5.9}$$

In table 1 we give some quantities of the empirical distribution of the terminal wealth $X(T)$, based on our simulation results, in three cases: a) when the wealth is controlled in order to minimize the mean square error and the stochastic nature of the claim intensity is not taken into account in the derivation of the optimal strategies and in the setting of the premiums, $\beta = 0, \mathbb{E}^{t,\lambda}[\int_t^T \lambda(s)ds] = 100(T-t)$; b) when the wealth is controlled in order to minimize the mean square error but the stochastic nature of the claim intensity is taken into account, $\beta = 0$; c) when the wealth is controlled according to the strategy (4.43) in order to minimize the variance and when the stochastic nature of the intensity is taken into account as well. We assume that $T = 2, r = 0,05, \theta = 0,05, P = 750, X(0) = 500$.

Table 1: Distribution of the wealth $X(2)$

	Case "a"	Case "b"	Case "c"
Mean value	660,727	661,462	750,000
Standard deviation	385,482	205,810	325,141
1st percentile	-369,200	120,497	-207,462
5th percentile	0,014	322,499	151,517
10th percentile	168,832	413,553	324,588
90th percentile	1135,731	924,365	1122,875
95th percentile	1252,909	989,441	1197,262
99th percentile	1434,568	1135,709	1342,399

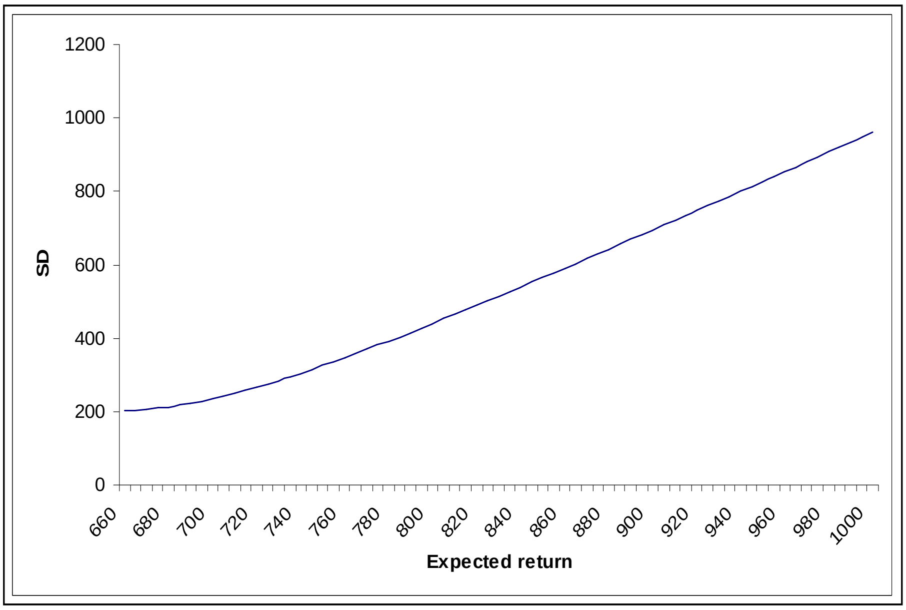

Figure 1: Capital market line

One should notice that the mean values of the wealth in the case "a" and the case "b" are the same, which can also be concluded based on (4.38). However, the standard deviation in the case "b" is lower than in the case "a", and the distribution of the terminal wealth is thicker-tailed in the case "a", compared with the case "b". This is the result of taking into account the changes in the claim intensity over time, as this decreases the variability of the cash flows. The terminal constraint and positive value of β lead to the strategy of investing higher amounts in the stock, see (4.43) and (4.28), and this explains the increase in the mean value, as well as in the standard deviation for the case "c", compared with the case "b". The Lagrange multiplier in our example is equal to $\beta = 1387$. It is also clear that the distribution of the terminal wealth in the case "c" has thicker tails than in the case "b". Finally, it is worth noticing that the standard deviation in the case "c" is lower than in the case "a", and that the distribution of the wealth has thicker tails in the case "a", compared with the case "c".

At the end, let us consider the efficient frontier and the efficient portfolios. In our example, the minimum expected return, which should be required by the insurer, is equal to $P^* = 665,75$. The capital market line can be found in figure 1. $\square$

6 Conclusions

In this paper we have dealt with a mean-variance portfolio selection for a non-life insurance company. We have assumed that insurance claims are generated according to a compound Cox process and that an asset return is driven by a Lévy process. We believe that this are very important extensions as far as integrated risk management is concerned. We have arrived at the smooth optimal value function, the optimal strategy and derived the efficient frontier in the Markowitz´s case.

We have only considered diffusion type claim intensity processes. This assumption is reasonable when dealing with large collective portfolios of independent risks. However, for an individual portfolio of a small size, a discontinuous claim intensity process seems to fit better. We would like to point out that one can choose a specific discontinuous claim intensity process, like a shot noise process or a more general Ornstein-Uhlenbeck process driven by a subordinator, apply the Feynman-Kac formula informally and then try to calculate the integrals from the representation explicitly by substituting the expected value function of the intensity. This might be tedious but if the result is a smooth function then all lemmas and theorems stated in this paper will also be satisfied.

Finally, we would like to refer the interested reader to the paper of Delong *et al* (2006), where mean-variance optimization problems for the accumulation phase in a defined benefit plan are considered, with the aim of hedging an annuity payment for a retiree in the presence of a stochastic mortality intensity.

7 Appendix

In order to prove the existence of a unique solution to a partial differential equation and its smoothness we use the theorem 1 from Heath, Schweizer (2000). Notice that in our case the coefficients are unbounded and the theorem cannot be applied directly. However, the problem lies only in taking limits under the expectation so it is sufficient to established uniform integrability, as pointed out in Heath, Schweizer (2000).

In the sequel, K denotes a constant whose value may change from one occurrence to the next.

Proof of lemma 3.1:
Fix $(t,\lambda)\in[0,T]\times(0,\infty)$. Define the sequence $\{\int_z^T \lambda^{z,y}(s)ds\}_{(z,y)\in U}$ where U is a compact set around (t,λ). Due to boundness of the continuous mapping $(z,y,s)\to\lambda^{z,y}(s)$ on compact sets, Lebesgue's dominated convergence theorem yields that

$$\lim_{(z,y)\to(t,\lambda)}\int_z^T \lambda^{z,y}(s)ds=\int_t^T \lambda^{t,\lambda}(s)ds,\quad \mathbb{P}-a.s. \tag{7.1}$$

Applying the Cauchy-Schwartz inequality we can arrive at

$$\mathbb{E}[|\int_z^T \lambda^{z,y}(s)ds|^2]\le K\mathbb{E}[\int_z^T (\lambda^{z,y}(s))^2ds]<\infty,\quad \forall(z,y)\in U. \tag{7.2}$$

This implies that the sequence $\{\int_z^T \lambda^{z,y}(s)ds\}_{(z,y)\in U}$ is uniformly integrable, which together with a.s. convergence (7.1), establishes the convergence in $L^1(\mathbb{P})$. We can now conclude that the mapping $(t,\lambda)\mapsto R(t,\lambda)$ is continuous. The rest of the lemma follows from Heath, Schweizer (2000). $\square$

Proof of lemma 4.1:

Define the function

$$\begin{aligned} B(t,\lambda) &= -\big(2(1-\alpha)P(T)+\beta\big)e^{(r-\delta)(T-t)} \\ &+ \ \mathbb{E}^{t,\lambda}\Big[\int_t^T\big\{-2\alpha\big(P(s)+R(s,\lambda(s))\big)+2\theta\mu_1 A(s)\lambda(s)\big\}e^{(r-\delta)(s-t)}ds\Big], \end{aligned} \tag{7.3}$$

for all $(t,\lambda)\in[0,T]\times(0,\infty)$. Due to the Markov property of the intensity process and the law of iterated expectations we have

$$\mathbb{E}^{t,\lambda}[R(s,\lambda(s))] = \mathbb{E}^{t,\lambda}\big[\mu_1\mathbb{E}[\int_s^T\lambda(u)du|\mathcal{F}_s]\big] = \mu_1\mathbb{E}^{t,\lambda}[\int_s^T\lambda(u)du], \tag{7.4}$$

for all $s\in[t,T]$. By applying Fubini's theorem, substituting (7.4) into (7.3), and changing the order of integration under the expected value in

$$\mathbb{E}^{t,\lambda}\Big[\int_t^T e^{(r-\delta)(s-t)}\int_s^T\lambda(u)duds\Big] = \mathbb{E}^{t,\lambda}\Big[\int_t^T\lambda(u)\int_t^u e^{(r-\delta)(s-t)}dsdu\Big], \tag{7.5}$$

we can arrive at (4.28). From the proof of lemma 3.1, it is clear that the sequence

$$\Big\{\int_z^T\Big(\theta A(s)e^{(r-\delta)(s-z)}-\alpha\frac{e^{(r-\delta)(s-z)}-1}{r-\delta}\Big)\lambda^{z,y}(s)ds\Big\}_{(z,y)\in U} \tag{7.6}$$

converges $\mathbb{P}$-a.s., as $(z,y)\to(t,\lambda)$, and is uniformly integrable, hence converges in $L^1(\mathbb{P})$. We can now conclude that the mapping $(t,\lambda)\mapsto B(t,\lambda)$ is continuous.

In order to prove that (7.3) is the probabilistic representation of a unique classical solution to the partial differential equation (4.23) we follow the proof of proposition 2.3 in Becherer, Schweizer (2005). Choose $\epsilon>0$ and consider the equation (4.23) on the time interval $[0,T-\epsilon]$. Notice that the function R is uniformly Hölder continuous on compact subsets of $[0,T-\epsilon]\times\bar{E}_n$. Based on theorem 1 in Heath, Schweizer (2000) we can conclude that the continuous mapping $B:[0,T-\epsilon]\times(0,\infty)\to\mathbb{R}$

$$\begin{aligned} B(t,\lambda) &= \mathbb{E}^{t,\lambda}[B(T-\epsilon,\lambda(T-\epsilon))e^{(r-\delta)(T-\epsilon-t)} \\ &+ \int_t^{T-\epsilon}\big\{-2\alpha\big(P(s)+R(s,\lambda(s))\big)+2\theta\mu_1A(s)\lambda(s)\big\}e^{(r-\delta)(s-t)}ds\big], \end{aligned} \tag{7.7}$$

is the unique classical solution to (4.23) on $[0,T-\epsilon]$. As ϵ is arbitrary, the existence of a classical solution to the partial differential equation (4.23) and its probabilistic representation (7.3) follow. The estimate (4.26) can be derived immediately from the representation (4.28).

Define the function

$$\begin{aligned} C(t,\lambda) &= (1-\alpha)P^2(T)+\beta P(T)+\mathbb{E}^{t,\lambda}\Big[\int_t^T\big\{\lambda(s)\mu_2A(s)+\theta\mu_1\lambda(s)B(s,\lambda(s)) \\ &\quad +\alpha\big(P(s)+R(s,\lambda(s))\big)^2-\frac{B^2(s,\lambda(s))}{4A(s)}\delta\big\}ds\Big], \end{aligned} \tag{7.8}$$

for all $(t,\lambda)\in[0,T]\times(0,\infty)$. First, apply the estimates (4.5) and (4.35), and conclude that

$$\begin{aligned}&\big|\lambda^{t,\lambda}(s)\mu_2 A(s)+\theta\mu_1\lambda^{t,\lambda}(s)B(s,\lambda^{t,\lambda}(s))\\ &+\ \alpha\big(P(s)+R(s,\lambda^{t,\lambda}(s))\big)^2\ -\frac{B^2(s,\lambda^{t,\lambda}(s))}{4A(s)}\delta\big|\\ &\le K\big(1+(\lambda^{t,\lambda}(s))^2+\mathbb{E}[\int_s^T(\lambda^{t,\lambda}(u))^2du|\mathcal{F}_s]\big),\end{aligned}\tag{7.9}$$

holds $\mathbb{P}$-a.s. for all $s\in[t,T]$. Then, let us consider the sequence of random variables $\{C_{z,y}\}_{(z,y)\in U}$, where

$$\begin{aligned}C_{z,y}\ &=\ \int_z^T\big\{\lambda^{z,y}(s)\mu_2A(s)+\theta\mu_1\lambda^{z,y}(s)B(s,\lambda^{z,y}(s))\\ &+\alpha\big(P(s)+R(s,\lambda^{z,y}(s))\big)^2-\frac{B^2(s,\lambda^{z,y}(s))}{4A(s)}\delta\big\}ds.\end{aligned}\tag{7.10}$$

It has been proved that the mappings $(t,\lambda)\mapsto R(t,\lambda)$ and $(t,\lambda)\mapsto B(t,\lambda)$ are both continuous, hence they are bounded on compact sets. Lebesgue's dominated convergence theorem yields that

$$\lim_{(z,y)\to(t,\lambda)}C_{z,y}=C_{t,\lambda},\quad \mathbb{P}-a.s..\tag{7.11}$$

The sequence is also uniformly integrable as

$$\begin{aligned}\mathbb{E}[|C_{z,y}|^2]\ &\le\ K\mathbb{E}\big[\int_z^T\big|\lambda^{z,y}(s)\mu_2A(s)+\theta\mu_1\lambda^{z,y}(s)B(s,\lambda^{z,y}(s))\\ &\quad+\alpha\big(P(s)+R(s,\lambda^{z,y}(s))\big)^2-\frac{B^2(s,\lambda^{z,y}(s))}{4A(s)}\delta\big|^2ds\big]\\ &\le\ K\mathbb{E}\left[\int_z^T\left|1+(\lambda^{z,y}(s))^2+\mathbb{E}[\int_s^T(\lambda^{z,y}(u))^2du|\mathcal{F}_s]\right|^2ds\right]\\ &\le\ K\mathbb{E}\left[\int_z^T\left(1+(\lambda^{z,y}(s))^4+\big(\mathbb{E}[\int_s^T(\lambda^{z,y}(u))^2du|\mathcal{F}_s]\big)^2\right)ds\right]\\ &\le\ K\mathbb{E}\left[\int_z^T\left(1+(\lambda^{z,y}(s))^4+\mathbb{E}[\int_s^T(\lambda^{z,y}(u))^4du|\mathcal{F}_s]\right)ds\right]\\ &=\ K\mathbb{E}\left[\int_z^T\left(1+(\lambda^{z,y}(s))^4+\int_s^T(\lambda^{z,y}(u))^4du\right)ds\right]\\ &\le\ K\mathbb{E}\left[1+\int_z^T(\lambda^{z,y}(s))^4ds\right]<\infty,\end{aligned}\tag{7.12}$$

for all $(z,y)\in U$, where we have applied Cauchy-Schwartz inequality, the estimate (7.9), Jensen's inequality for conditional expectations, Fubini's theorem, the law of iterated expectations and at last, we have changed the order of integration under the expected value in

$$\mathbb{E}\big[\int_z^T\int_s^T(\lambda^{z,y}(u))^4duds\big]=\mathbb{E}\big[\int_z^T\int_z^u(\lambda^{z,y}(u))^4dsdu\big].\tag{7.13}$$

We can conclude that the sequence $\{C_{z,y}\}_{(z,y)\in U}$ converges in $L^1(\mathbb{P})$, and hence, the mapping $(t,\lambda)\mapsto C(t,\lambda)$ is continuous. In order to prove the existence of a smooth solution to the partial differential equation (4.24) and its probabilistic representation (7.7) one should now follow, as previously, Heath, Schweizer (2000), theorem 1, and Becherer, Schweizer (2005), proposition 2.3. The estimate (4.27) can be derived from (7.9) in the same way as the estimate (7.12). □

Proof of lemma 4.2:

The existence and uniqueness follows from the general theory of stochastic differential equations driven by discontinuous semimartingales in the case of functional Lipschitz coefficients, see Protter (2005), theorem V.7, for details. To arrive at the second part of the lemma, one should apply well-known techniques from the theory of differential equations. Define the sequence of stopping times $\tau_m = \inf\{s\in(0,t], |X^{\hat{\pi}}(s)-x|>m\}$ and show that

$$\mathbb{E}^{0,x,\lambda}\big[|X^{\hat{\pi}}(t)|^4\mathbf{1}\{\tau_m>t\}\big]\leq K\big(1+\int_0^t\mathbb{E}^{0,x,\lambda}\big[|X^{\hat{\pi}}(s)|^4\mathbf{1}\{\tau_m>s\}\big]du\big),\qquad(7.14)$$

for an arbitrary $x\in\mathbb{R},\lambda\in(0,\infty),t\in[0,T]$ and some constant $K<\infty$ which depends on (x,t,λ). This can be derived by applying an estimate of moments of stochastic integrals of predictable processes with respect to Lévy processes, see Protter (2005), as well as the estimate techniques similar to those from the proof of lemma 4.1. The result follows from Gronwall's inequality and Fatou's lemma by taking limit $m\to\infty$. □

Proof of theorem 4.1:

Fix $(t,x,\lambda)\in[0,T)\times\mathbb{R}\times(0,\infty)$ and ϵ such that $0<\epsilon<T-t$. Define a sequence of stopping times $t_n = \inf\{s\in(t,T]; |X^{\pi}(s)-x|+|\lambda(s)-\lambda|>n\}$. We apply Itô's formula for discontinuous semimartingales to a function v on the time interval

$[t, t_n \wedge (T-\epsilon)]$, see Applebaum (2004), theorem 4.4.10. We arrive at

$$\begin{aligned}
&\mathbb{E}^{t,x,\lambda}\big[v(t_n \wedge (T-\epsilon), X^\pi(t_n \wedge (T-\epsilon)), \lambda(t_n \wedge (T-\epsilon))) - v(t,x,\lambda)\big] \\
&= \mathbb{E}^{t,x,\lambda}\Big[\int_{(t,T-\epsilon]} \mathbf{1}\{t_n \geq s\}\frac{\partial v}{\partial t}(s, X^\pi(s-), \lambda(s))ds \\
&\quad + \int_{(t,T-\epsilon]} \mathbf{1}\{t_n \geq s\}\frac{\partial v}{\partial x}(s, X^\pi(s-), \lambda(s))dX^\pi(s) \\
&\quad + \int_{(t,T-\epsilon]} \mathbf{1}\{t_n \geq s\}\frac{\partial v}{\partial \lambda}(s, X^\pi(s-), \lambda(s))d\lambda(s) \\
&\quad + \frac{1}{2}\int_{(t,T-\epsilon]} \mathbf{1}\{t_n \geq s\}\frac{\partial^2 v}{\partial x^2}(s, X^\pi(s-), \lambda(s))\pi^2(s)\sigma^2 ds \\
&\quad + \frac{1}{2}\int_{(t,T-\epsilon]} \mathbf{1}\{t_n \geq s\}\frac{\partial^2 v}{\partial \lambda^2}(s, X^\pi(s-), \lambda(s))b^2(s,\lambda(s)) ds \\
&\quad + \sum_{\triangle L(s)\neq 0, t<s\leq T-\epsilon} \mathbf{1}\{t_n \geq s\}\big(v(s, X^\pi(s-) + \pi(s)\triangle L(s), \lambda(s)) \\
&\quad - v(s, X^\pi(s-), \lambda(s)) - \pi(s)\triangle L(s)\frac{\partial v}{\partial x}(s, X^\pi(s-), \lambda(s))\big) \\
&\quad + \sum_{\triangle C(s)\neq 0, t<s\leq T-\epsilon} \mathbf{1}\{t_n \geq s\}\big(v(s, X^\pi(s-) - \triangle C(s), \lambda(s)) \\
&\quad - v(s, X^\pi(s-), \lambda(s)) + \triangle C(s)\frac{\partial v}{\partial x}(s, X^\pi(s-), \lambda(s))\big)\Big].
\end{aligned} \tag{7.15}$$

Notice that we can disjoint the jumps $\triangle L$ and $\triangle C$ of the process X^π, due to the independence of the processes L and C. The formula (7.15) can be rewritten as

$$\begin{aligned}
&\mathbb{E}^{t,x,\lambda}\big[v(t_n \wedge (T-\epsilon), X^\pi(t_n \wedge (T-\epsilon)), \lambda(t_n \wedge (T-\epsilon))) - v(t,x,\lambda)\big] \\
&= \mathbb{E}^{t,x,\lambda}\Big[\int_t^{T-\epsilon} \mathbf{1}\{t_n \geq s\}\big(\frac{\partial v}{\partial t}(s, X^\pi(s-), \lambda(s)) + \mathcal{L}_F^{\pi(s)}v(s, X^\pi(s-), \lambda(s)) \\
&\quad + \mathcal{L}_I v(s, X^\pi(s-), \lambda(s)) + \mathcal{L}_C v(s, X^\pi(s-), \lambda(s))\big)ds \\
&\quad + \int_t^{T-\epsilon} \mathbf{1}\{t_n \geq s\}\frac{\partial v}{\partial x}(s, X^\pi(s-), \lambda(s))\pi(s)\sigma(s)dW(s) \\
&\quad + \int_t^{T-\epsilon} \mathbf{1}\{t_n \geq s\}\frac{\partial v}{\partial \lambda}(s, X^\pi(s-), \lambda(s))b(s,\lambda(s))d\bar{W}(s) \\
&\quad + \int_{(t,T-\epsilon]}\int_{z>-1} \mathbf{1}\{t_n \geq s\}\big(v(s, X^\pi(s-) + \pi(s)z, \lambda(s)) \\
&\quad - v(s, X^\pi(s-), \lambda(s))\big)\tilde{M}(ds \times dz) \\
&\quad + \int_{(t,T-\epsilon]}\int_0^\infty \mathbf{1}\{t_n \geq s\}\big(v(s, X^\pi(s-) - z, \lambda(s)) \\
&\quad - v(s, X^\pi(s-), \lambda(s))\big)\tilde{N}(ds \times dz)\Big].
\end{aligned} \tag{7.16}$$

The expected values of the stochastic integrals are equal to zero, see Protter (2005) and Appleabaum (2004) for the theory of the stochastic integration. Notice that the last integral in (7.16) is a stochastic integral with respect to a general martingale, not a compensated Poisson measure. The next steps are rather standard. Taking the limit $n \to \infty, \epsilon \to 0$, the inequality (4.17) and the equality (4.20) are established with Fatou's lemma and Lebesgue's dominated convergence theorem. □

We would like to point out that our localization procedure is crucial as we can omit some of the conditions stated for example in Øksendal, Sulem (2005), theorem 3.1, in the case of Lévy-diffusion processes. Without this localization procedure, it would be hard and would require stronger assumptions, to check the general conditions of the verification theorem.

Proof of theorem 4.2:

The investment strategy (4.43) is admissible, as it is a square integrable, predictable process, such that the optimal fund process $X^{\hat{\pi}}$ is unique, see (4.36) and lemma 4.2. Our solution of the Hamilton-Jacobi-Bellman equation, in the form of $A(t)x^2 + B(t,\lambda)x + C(t,\lambda)$, is smooth as required. The conditions (4.11), (4.12) and (4.18) are clearly satisfied, by the method of constructing the solution, and (4.21) is indeed the minimizer of the quadratic function in π. It is straightforward to prove, in the same manner as (4.36), that $\mathbb{E}^{0,\lambda}[|B(t,\lambda(t))|^4] < \infty$. This condition together with the finiteness of the fourth moment of $X^{\hat{\pi}}$, established in lemma 4.2, and assumptions (A3), (B1), (B5), imply that (4.13), (4.14) and (4.15) are satisfied. In order to prove uniform integrability, notice that for any $\mathbb{F}$-stopping time τ the following inequality holds

$$\begin{aligned}
&\mathbb{E}^{0,x,\lambda}[|v(\tau, X^{\hat{\pi}}(\tau), \lambda(\tau))|^2] \\
&\quad \leq K\mathbb{E}^{0,x,\lambda}[|X^{\hat{\pi}}(\tau)|^4 + |B(\tau,\lambda(\tau))|^4 + |C(\tau,\lambda(\tau))|^2].
\end{aligned} \tag{7.17}$$

Similarly, as in the proof of lemma 4.2, one can show that

$$\begin{aligned}
&\mathbb{E}^{0,x,\lambda}\big[|X^{\hat{\pi}}(\tau)|^4\big] \\
&\quad \leq K\big(1 + \int_0^T \mathbb{E}^{0,x,\lambda}\big[|X^{\hat{\pi}}(s)|^4 \mathbf{1}\{s \leq \tau\}\big]ds\big) \\
&\quad \leq K\big(1 + \int_0^T \mathbb{E}^{0,x,\lambda}\big[|X^{\hat{\pi}}(s)|^4\big]ds\big) < \infty.
\end{aligned} \tag{7.18}$$

Due to the estimate (4.35), the strong Markov property of the intensity process and the law of iterated expectations we can also arrive at

$$\begin{aligned}
&\mathbb{E}^{0,\lambda}[|B(\tau,\lambda(\tau))|^4] \\
&\quad \leq K\big(1 + \mathbb{E}^{0,\lambda}\big[\mathbb{E}\big[\int_\tau^T (\lambda(s))^4 ds|\mathcal{F}_\tau\big]\big]\big) \\
&\quad \leq K(1 + \mathbb{E}^{0,\lambda}\big[\int_0^T (\lambda(s))^4 ds\big]) < \infty.
\end{aligned} \tag{7.19}$$

The finiteness of the third term in (7.17) is proved in the same way. We can now conclude that the strategy (4.43) is optimal for the optimization problem (4.3). The parameter β is chosen such that the constraint on the expected value of the surplus is satisfied and, as a result, the value function for (4.3) at time 0 is equal to the cost (4.1).

□

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